Cosmic numbers and Physmatics
Números cósmicos y Fismática

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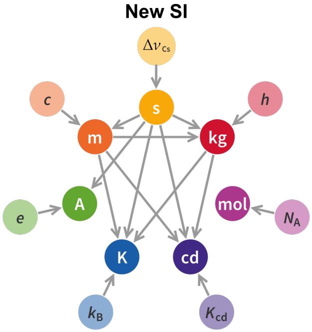
2020
Earth planet
Milky Way Galaxy

Contenido

Magnitudes

Magnitudes: basic and derived/básicas y derivadas

Magnitud es todo aquello que podemos **medir**. Hay magnitudes cuantitativas y cualitativas. También, en teorías modernas, puede haber magnitudes no observables. En todo sistema de unidades hay magnitudes básicas o fundamentales, y derivadas (combinación de dos o más de una magnitud básica).



S.I.: Sistema Internacional

El S.I. es el sistema de unidades en el que se define como magnitudes básicas: la longitud (L), el tiempo(T), la cantidad de materia o masa (M), la cantidad de sustancia (n), la temperatura absoluta(Θ), la intensidad luminosa (J, I_L) y la intensidad de corriente eléctrica (I).

Tiempo/Time

Tiempo es magnitud base en el S.I. Su símbolo dimensional es T. La unidad base es el segundo, definido como 9192631770 ciclos de la radiación de la transición hiperfina no perturbada fundamental del átomo de cesio-133. Matemáticamente:

$$1Hz = \frac{\Delta f(Cs-133)}{9192631770}s^{-1} \leftrightarrow 1s = \frac{9192631770}{\Delta\nu(Cs-133)} \quad (1)$$

Proton lifetime is about $t(p^+) \geq 10^{34} yrs = 10^{41} s$. Proton decay is expected naturally at some point between 10^{45}yrs or 10^{122}yrs.

7 magn. básicas(II)/ 7 basic magnitudes: Longitud/Length

Longitud/Length

Longitud es magnitud base en el S.I. Su símbolo dimensional es L. La unidad base es el metro definido como la distancia que recorre la luz en 1/299792458 segundos. Equivalentemente, se define como el valor numérico fijo de la velocidad de la luz en el vacío, expresando la velocidad en metros por segundo, y el segundo definido relativo a la definición de la frecuencia $\Delta(Cs-133)$. Esto da como valor exacto $c = 299792458 m/s$, mientras que el metro queda definido en función de c y de $\Delta f(^{133}Cs)$:

$$1m = \frac{c}{299792458}s = \frac{9192631770}{299792458}\frac{c}{\Delta f(^{133}Cs)} \approx 30{,}663319\frac{c}{\Delta f(^{133}Cs)} \quad (2)$$

7 magn. básicas(III)/ 7 basic magnitudes: Masa/Mass

Masa/Mass

Masa es magnitud base de cantidad de materia en el S.I. Su símbolo dimensional es M. La unidad base es el kilogramo definido usando la constante de Planck $h = 6{,}62607015 \cdot 10^{-34}$ como fija en unidades de $J \cdot s$ ó J/Hz, o bien $kg \cdot m^2/s$. Esto da como valor exacto de un kilogramo:

$$1kg = \frac{h}{6{,}62607015 \cdot 10^{-34}} \frac{s}{m^2} = \frac{299792458^2}{(6{,}62607015 \cdot 10^{-34})(9192631770)} \frac{h\Delta f}{c^2} =$$

$$= 1.4755214 \cdot 10^{40} \frac{h\Delta f_{Cs}}{c^2} (3)$$

Cantidad de sustancia/Amount of substance

antidad de sustancia es magnitud base del S.I. Su símbolo dimensional es n. La unidad base es el mol (mol), definido como la cantidad de sustancia que contiene exactamente una cantidad igual a la constante de Avogadro N_A, fijada al valor $N_A = 6{,}02214076 \cdot 10^{23} mol^{-1}$. De aquí, un mol se define mediante el factor de conversión siguiente:

$$1mol = \frac{6{,}02214076 \cdot 10^{23}}{N_A} \quad (4)$$

La cantidad de sustancia es una medida del número de entidades elementales en cualquier pedazo de materia. Puede ser de átomos, moléculas, iones, electrones o cualquier otra partícula o grupo de partículas que se especifique.

Temperatura absoluta/Absolute temperature

Temperatura absoluta es una magnitud base en el S.I. Su símbolo dimensional es T ó Θ. La unidad base es el grado kelvin K definido usando la constante de Boltzmann, expresada en J/K como $k_B = 1{,}380649 \cdot 10^{-23}$ como fija, o bien en unidades dimensionales del S.I. como $kg \cdot m^2 \cdot s^{-2} \cdot K^{-1}$. Entonces, el kelvin (grado kelvin) se define mediante el factor de conversión:

$$1K = \frac{1{,}380649 \cdot 10^{-23}}{k_B} kg \cdot m^2 \cdot s^{-2} =$$

$$= \frac{1{,}380649 \cdot 10^{-23}}{(6{,}62607015 \cdot 10^{-34})(9192631770)} \frac{h\Delta f}{k_B} \approx 2{,}2666653 \frac{h\Delta f_{Cs}}{k_B} \quad (5)$$

Electric current intensity/Intensidad de corriente eléctrica

Intensidad de corriente eléctrica es magnitud base en el S.I. Su símbolo dimensional es I. La unidad base es el amperio A definido usando la constante definida por la carga elemental del electrón $Q(e) = e = 1{,}602176634 \times 10^{-19} C(A \cdot s)$ como fija. Entonces, el amperio se define mediante el factor de conversión:

$$1A = \frac{e}{1{,}602176634 \times 10^{-19}} s^{-1} = \frac{e\Delta f(Cs - 133)}{(1{,}602176634 \times 10^{-19})(9192631770)} \approx$$

$$\approx 6{,}789687 \cdot 10^{8} e\Delta f_{Cs} (6)$$

Intensidad luminosa/Luminous intensity

La intensidad luminosa en una dirección dada es una magnitud base del S.I. Su símbolo dimensional es I_L, o también I_ν ó $\mathcal{J}$. La unidad base de intensidad luminosa es la candela cd, definida como la cantidad que, tomando como valor numérico fijo la eficacia luminosa de la radiación monocromática de frecuencia 540THz, K_{cd}, ésta es 683 expresada en unidades de lúmens por vatio, $lm \cdot W^{-1}$, o bien en candelas por estereoradián entre vatio $cd \cdot sr \cdot W^{-1}$, o también $cd \cdot sr \cdot kg^{-1} \cdot m^{-2} \cdot s^3$, donde el kilogramo, el metro, el segundo se definen mediante las constantes $h, c, \Delta f_{Cs}$. Con esta definición, usando $K_{cd}, h, c, \Delta f_{Cs}$ a:

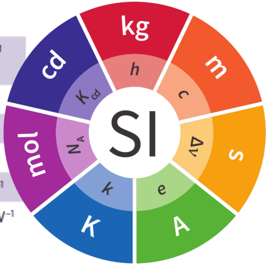
$$1cd = \frac{K_{cd}}{683}\frac{kg \cdot m^2}{s^3 \cdot sr} = \frac{K_{cd} h \cdot [\Delta f_{Cs}]^2}{(6{,}62607015 \cdot 10^{-34})(9192631770)^2 683} \approx$$

$$\approx 2{,}61483010 \times 10^{10} K_{cd} h [\Delta f_{Cs}]^2 (7)$$

El nuevo S.I./The new S.I.

Defining constant	Symbol	Numerical value	Unit
hyperfine transition frequency of caesium	$\Delta\nu_{Cs}$	9 192 631 770	Hz
speed of light in vacuum	c	299 792 458	m s^{-1}
Planck constant	h	$6.626\ 070\ 15 \times 10^{-34}$	J s
elementary charge	e	$1.602\ 176\ 634 \times 10^{-19}$	C
Boltzmann constant	k	$1.380\ 649 \times 10^{-23}$	J K^{-1}
Avogadro constant	N_A	$6.022\ 140\ 76 \times 10^{23}$	mol^{-1}
luminous efficacy	K_{cd}	683	lm W^{-1}

Table 1: The seven defining constants of the SI, and the seven corresponding symbols, numerical values, and units

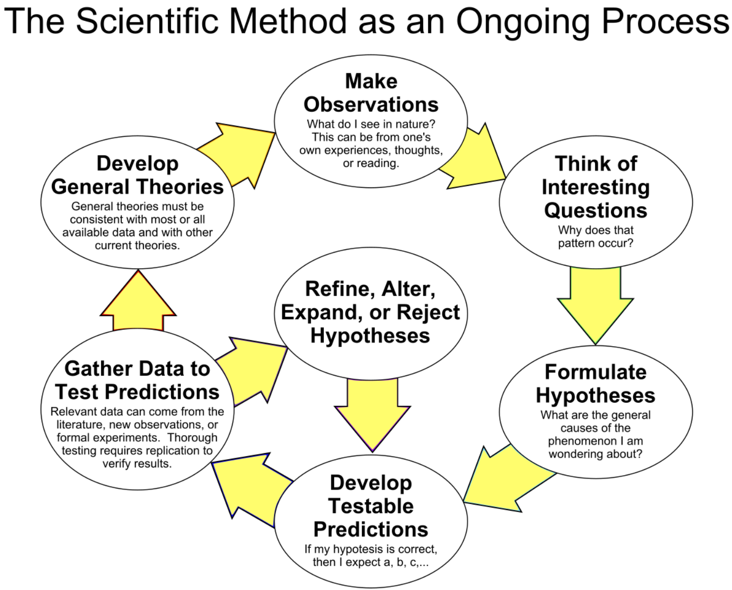


Contenido

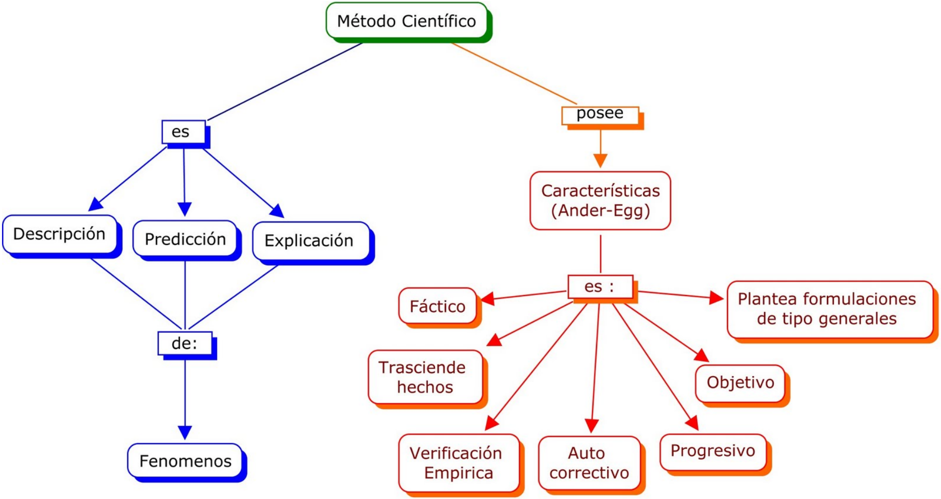
Scientific Method

El método científico/Scientific method

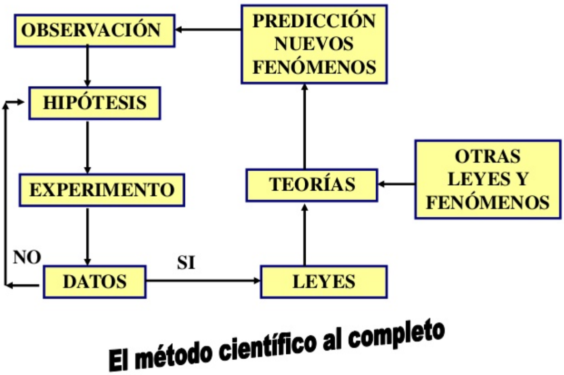
A (cyclical, iterative, systematic) method/procedure to acquire, gather, organize, check (verify or refute)and test, conserve (preserve) and transmit (communicate) knowledge (both in form of data or organized abstract data/axioms/propositions) or more generally useful important information built from experience, reason and thought. Based upon **curiosity** and the *will to know*. Use Mathematics as its language.
[Método o proceso cíclico, sistemático e iterativo para adquirir, reunir, organizar, comprobar y verificar, conservar (preservar) y transmitir(comunicar) conocimiento (en forma de datos o datos abstractos, axiomas, proposiciones), o más generalmente información útil importante a partir de la experiencia, razón y el pensamiento. Basada en la curiosidad y voluntad de saber(y entender). Usa las Matemáticas como su lenguaje.]


The Scientific Method as an Ongoing Process
Make Observations
What do I see in nature? This can be from one's own experiences, thoughts, or reading.
Think of Interesting Questions
Why does that pattern occur?
Formulate Hypotheses
What are the general causes of the phenomenon I am wondering about?
Develop Testable Predictions
If my hypotesis is correct, then I expect a, b, c,...
Gather Data to Test Predictions
Relevant data can come from the literature, new observations, or formal experiments. Thorough testing requires replication to verify results.
Develop General Theories
General theories must be consistent with most or all available data and with other current theories.
Refine, Alter, Expand, or Reject Hypotheses

Scientific Method(III)/Método científico(III)



Scientific Method(IV)/Método científico(IV)



Keywords(palabras clave): hipótesis, axioma, ley, principio, postulado, teoría, paradigma, modelo, realidad, big data, AI, estadística, revolución científica, Machine Learning, simulación, emulación, lógica, gedanken experiment, experimento, heurística, algoritmo, prueba y error.

Contenido

Potencias de 10 y notación científica

- *La notación científica es básica para expresar resultados y cuantificar magnitudes,*

Scientic notation

$$X = q.x_1x_2x_3\ldots x_p\ldots\cdot 10^{\pm n} \quad (8)$$

where $q\neq 0$ and $x_1, x_2, \ldots, x_p, n$ are base 10 digits (positive integer numbers)/ donde $q\neq 0$ and $x_1, x_2, \ldots, x_p, n$ son dígitos de base 10 (números enteros positivos)

  - Podría usarse en el fondo cualquier numeración, cualquier sistema numérico de base B, pero usamos la base 10.

- La notación permite números grandes o gigantescos, y también números muy, muy, pero que muy pequeños.

S.I. prefixes/prefijos del S.I.

PEZY-fazy rules! PezYosos-FaZyles prefijos/prefixes

METRIC PREFIXES			
Power of Ten	Exponential Notation	Metric Prefix	Abbreviation
septillion	10^{24}	yotta	Y
sextillion	10^{21}	zetta	Z
quintillion	10^{18}	exa	E
quadrillion	10^{15}	peta	P
trillion	10^{12}	tera	T
billion	10^{9}	giga	G
million	10^{6}	mega	M
thousand	10^{3}	kilo	k
hundred	10^{2}	hecto	h
ten	10^{1}	deca	da
tenth	10^{-1}	deci	d
hundredth	10^{-2}	centi	c
thousandth	10^{-3}	milli	m
millionth	10^{-6}	micro	μ
billionth	10^{-9}	nano	n
trillionth	10^{-12}	pico	p
quadrillionth	10^{-15}	femto	f
quintillionth	10^{-18}	atto	a
sextillionth	10^{-21}	zepto	z
septillionth	10^{-24}	yocto	y

Extras (non-accepted yet)

Table 2. Proposed prefixes for powers of ten larger than 24 and smaller than –24.

Power of ten	Prefix	Symbol	Origin
–27	xenno	x	Gr, ennea, nine
–30	weko	w	Gr, deka, ten
–33	vendeko	v	Gr, hendeka, eleven
–36	udeko	u	Gr, dodeka, twelve
27	xenta	X	Gr, ennea, nine
30	wekta	W	Gr, deka, ten
33	vendekta	V	Gr, hendeka, eleven
36	udekta	U	Gr, dodeka, twelve

S.I. prefixes/prefijos del S.I.(III)

Q1. Familiarization with prefixes and abbreviations.

10^n	Prefix	Abbreviation	10^n	Prefix	Abbreviation
10^0					
10^3	kilo-	k	10^{-3}	milli-	m
10^6	mega-	M	10^{-6}	micro-	μ
10^9	giga-	G	10^{-9}	nano-	n
10^{12}	tera-	T	10^{-12}	pico-	p
10^{15}	peta-	P	10^{-15}	femto-	f
10^{18}	exa-	E	10^{-18}	atto-	a
10^{21}	zetta-	Z	10^{-21}	zepto-	z
10^{24}	yotta-	Y	10^{-24}	yocto-	y

Suppose, $x = ay^4$ where $a = 2.00$ ng/Pm. Determine the value of y when x = 16.0 (Zg fm^2)/(ms^3). Express the result in scientific notation and simplify the units.

(Hint: Refer to the table above, and note that SI prefixes are never used to multiply powers of units. For example, the abbreviation cm^2 means $(10^{-2}$ m$)^2$, not 10^{-2} m^2, and ns^{-1} is 1/ns or 10^9 s^{-1}, not 10^{-9}s^{-1}. Also note m that can stand for meter or for milli, depending on the context.)

Contenido

El googol y el ajedrez (Googol and chess)

Google viene de googol

El googol y el googolplex

$$10^{100} = \text{Googol} \qquad 10^{10^{100}} = 10^{\text{googol}} = \text{Googolplex} \tag{9}$$

Movimientos de ajedrez en una partida estándar:

Chess moves standard game

$$N_C(SG) = 10^{120}$$

Demo

First move: ~ 30. Second move $\sim 30 \cdot 30$. Supposing 40 moves per player, you have $40 \cdot 2$ total moves, and $\sim 30^{80} = 3^{80} \cdot 10^{80} \simeq 10^{40}10^{80} = 10^{120}$ total moves. Q.E.D.

Cubos de Rubik(I)

Un ejemplo de aplicaciones de notación científica y números son los cubos de Rubik y sus variantes hiperdimensionales.

Factorial

$$n! = n(n-1)\cdots 3 \cdot 2 \cdot 1 \tag{10}$$

Configuraciones de un cubo de Rubik
$3^3 = 3 \times 3 \times 3$:

3x3x3 Rubik cube

$$N_R(3^3) = \frac{3^7 \cdot 12! 2^{11}}{2} \simeq 4{,}33 \cdot 10^{19}$$

Cubo 3x3x3 desarmado

El cubo de Rubik 3^3 desarmado tiene $N_D = 12 N_R$, es decir:

$$N_D = 8! 3^8 \cdot 12! 2^{12} = \frac{12! 8! 2^{12} \cdot 3^8}{2 \cdot 2 \cdot 3}$$

$$N_D \simeq 5{,}19 \cdot 10^{20} \sim 10^{20}$$

Cubos de Rubik(II)

Posibles configuraciones de un cubo de Rubik $2^3 = 2 \times 2 \times 2$:

2x2x2 Rubik cube

$$N_C(2^3) = \frac{8!3^7}{24} = 7!3^6 = 3674160$$

$$N_C(2^3) \simeq 3{,}67 \cdot 10^6$$

Posibles configuraciones del cubo de Rubik $4^3 = 4 \times 4 \times 4$

Cubo 4x4x4

$$N_C(4^3) = \frac{8!3^7 24!^2}{24^7} \approx 7{,}40 \cdot 10^{45}$$

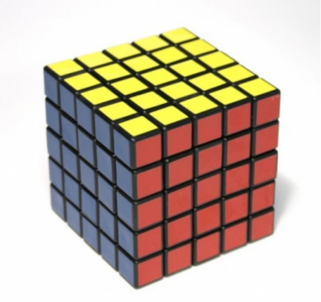
Cubos de Rubik(III)

Posibles configuraciones de un cubo de Rubik $5^3 = 5 \times 5 \times 5$:

5x5x5 Rubik cube

$$N_C(5^3) = \frac{8!3^7 12! 2^{10} 24!^3}{24^{12}}$$

$$N_C(5^3) \simeq 2{,}83 \cdot 10^{74}$$

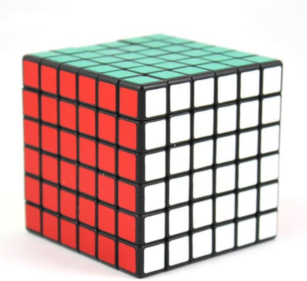


Posibles configuraciones del cubo de Rubik 6x6x6

Cubo 6x6x6

$$N_C(6^3) = \frac{8!3^7 24!^6}{24^{25}}$$

$$N_C(6^3) \simeq 1{,}57 \cdot 10^{116}$$



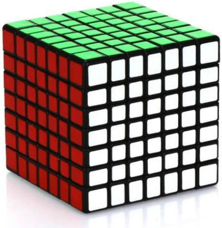
Cubos de Rubik(IV)

Posibles configuraciones de un cubo de Rubik $7^3 = 7 \times 7 \times 7$:

7x7x7 Rubik cube

$$N_C(7^3) = \frac{8!3^7 12! 2^{10} 24!^8}{24^{36}}$$

$$N_C(7^3) \simeq 1{,}95 \cdot 10^{160}$$

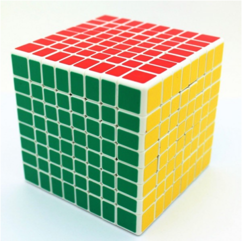


Posibles configuraciones del cubo de Rubik 6x6x6

Cubo 8x8x8

$$N_C(8^3) = \frac{8!3^7 24!^{12}}{24^{55}}$$

$$N_C(8^3) \simeq 3{,}52 \cdot 10^{217}$$



Megaminx and alike

Configuraciones de un megaminx:

Megaminx

$$N_M = 20! \cdot 3^{19} \cdot 30! \cdot 2^{27}$$

$$N_M \simeq 1{,}01 \cdot 10^{68}$$

or reducing by 2^{14}. The full number is 100 669 616 553 523 347 122 516 032 313 645 505 168 688 116 411 019 768 627 200 000 000 000.

$$N_M = 20! \cdot 3^{19} \cdot 30! \cdot 2^{13}$$

$$N_M \simeq 6{,}14 \cdot 10^{63}$$

Para el gigaminx, teraminx, petaminx, examinx, zettaminx y yottaminx está la fórmula

n-Minx

$$N_n = \frac{30!20!60!^{n^2-1}2^{28-n} \cdot 3^{19}}{5!^{12n(n-1)}}$$

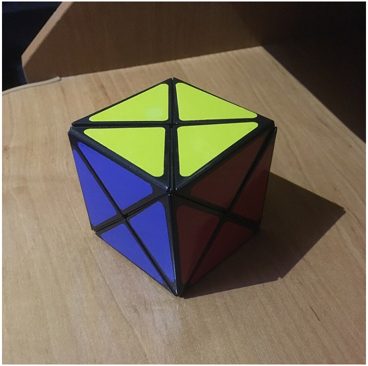
que produce $N \simeq 3{,}65 \cdot 10^{263}$ para los gigaminx,
$N \simeq 1{,}15 \cdot 10^{573}$ el teraminx,
$N \simeq 3{,}16 \cdot 10^{996}$ el petaminx,
$N \simeq 7{,}58 \times 10^{1533}$ el examinx,
$N \simeq 1{,}58 \times 10^{2185}$ el zettaminx,
$N \simeq 2{,}87 \times 10^{2950}$ el yottaminx,
not including correction factors.

Other cubes

Dino cube

$$N_D = \frac{2^{11}}{2} = 19958400$$

$$N_D \simeq 2{,}0 \cdot 10^7$$



Para el Skewb puzzle (kilominx):

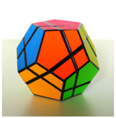
Kilominx

$$N_K = \frac{4!6!2^5}{4} = 138240 \simeq 1{,}4{\cdot}10^5$$

El Skewb Ultimate tiene

$$N_K(U) = \frac{6!2^5 4!3^6}{4} = 100776960$$

$$N_K(U) \simeq 1{,}01 \cdot 10^8$$



Other cubes(II)

Pyraminx crystal

$$N_D = \frac{30!2^{27}20!3^{19}}{60}$$

$$N_D \simeq 1{,}68 \cdot 10^{66}$$

Tuttminx

$$N_D = \frac{60!60!30!2^{29}}{8}$$

$$N_D \simeq 1{,}23 \cdot 10^{204}$$

Other cubes(III)

Impossiball

$$N_D = \frac{20!3^{19}}{120}$$

$$N_D \simeq 2{,}36 \cdot 10^{25}$$

Alexander's Star

$$N_D = \frac{30!2^{15}}{120}$$

$$N_D \simeq 7{,}24 \cdot 10^{34}$$

Dogic cubes

10-color Dogic

$$N_D = \frac{59!20!}{2^{11}6!^{10}}$$

$$N_D \simeq 4{,}4 \cdot 10^{66}$$

The precise figure is 4 400 411 583 858 825 100 777 127 453 704 140 502 784 413 155 112 522 644 357 120 000 000.

12-color Dogic

$$N_D = \frac{5912013^{19}}{2 \cdot 5!^{12}}$$

$$N_D \simeq 2{,}20 \cdot 10^{82}$$

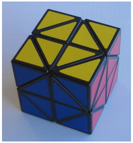
The precise figure is 21 991 107 793 244 335 592 538 616 581 443 187 569 604 232 889 165 919 156 829 382 848 981 603 083 878 400 000.

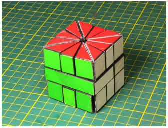
Other cubes(IV)

Helicopter cube

$$N_D = \frac{7!3^6 6!^4}{2}$$

$$N_D \simeq 4{,}94 \cdot 10^{17}$$





Square One puzzle

$$N_D = 170 \cdot 2 \cdot 8!^2$$

$$N_D = 552738816000$$

$$N_D \simeq 5{,}52 \cdot 10^{11}$$

Square Two puzzle

$$N_D = \frac{24!}{72}$$

$$N_D \simeq 8{,}62 \cdot 10^{21}$$

Higher dimensional Rubik cubes(I): 4d

Rubik 2x2x2x2 tesseract/hypercube

$$N_D(2^4) = \frac{15!}{2}\left(\frac{4!}{2}\right)^{14} \cdot 4 \sim 10^{28}$$

Rubik 3x3x3x3 hypercube/tesseract

$$N_C(3^4) = \frac{24!32!}{2}\frac{16!}{2}2^{23}(3!)^{31}3\left(\frac{4!}{2}\right)^{15} \cdot 4 \sim 10^{120}$$

Rubik 4x4x4x4 tesseract/hypercube

$$N_D(4^4) = \frac{15!}{2}\left(\frac{4!}{2}\right)^{14} \cdot 4\frac{64!}{2}3^{63}\frac{96!2}{2(4!)^{24}}\frac{2^{95}64!2}{2(8!)^{8}} \sim 10^{334}$$

Higher dimensional Rubik cubes(II): from 4d to 5d

Rubik 5x5x5x5 tesseract/hypercube

$$N=\frac{48!96!64!}{(6!)^8(12!)^8(8!)^8}\cdot\frac{24!32!(3!)^{31}2^{23}}{2}\frac{64!3^{63}16!}{2}\left(\frac{4!}{2}\right)^{15}\frac{2^{190}96!^2 4}{(4!)^{48}}\sim 10^{701}$$

Rubik 2x2x2x2x2 penteract/hypercube

$$N=\frac{31!60^{31}}{2}\sim 10^{89}$$

Rubik 3x3x3x3x3 penteract/hypercube

$$N=\frac{32!}{2}\cdot 60^{32}\cdot\frac{80!}{2}\cdot\frac{24^{80}}{2}\cdot\frac{40!\cdot 80!}{2}\cdot\frac{6^{80}}{2}\cdot\frac{2^{40}}{2}\simeq 7{,}02\cdot 10^{560}\sim 10^{561}$$

Note that the trivial plane Rubik 3x3 cube has 24=4! possible positions.

Rubik 5d hypercubes

$$N(4^5) = \frac{31!}{2}\cdot 60^{31}\cdot\frac{160!}{2}\cdot\frac{12^{160}}{3}\cdot\frac{320!}{24^{80}}\cdot\frac{6^{320}}{2}\cdot\frac{320!}{8!^{40}}\cdot\frac{2^{320}}{2}\cdot\frac{160!}{16!^{10}}\sim 10^{2075}$$

$$N(5^5) = \begin{cases}\frac{32!}{2}\cdot 60^{32}\cdot\frac{80!}{2}\cdot\frac{24^{80}}{2}\cdot\frac{160!}{2}\cdot\frac{12^{160}}{3}\cdot\frac{40!\cdot 80!}{2}\cdot\frac{6^{80}}{2}\cdot\frac{2^{40}}{2}\cdot\frac{320!}{24^{80}}\cdot\\ \cdot\frac{6^{320}}{2}\cdot\frac{320!}{24^{80}}\cdot\frac{6^{320}}{2}\frac{240!}{(6!)^{40}}\cdot\frac{2^{240}}{2}\cdot\frac{320!}{(8!)^{40}}\cdot\frac{2^{320}}{2}\cdot\frac{480!}{(12!)^{40}}\cdot\frac{2^{480}}{2}\cdot\frac{80!}{(8!)^{10}}\\ \cdot\frac{160!}{(16!)^{10}}\cdot\frac{240!}{(24!)^{10}}\cdot\frac{320!}{(32!)^{10}}\sim 10^{5267}\end{cases}$$

Higher dimensional Rubik cubes(IV): 5d and beyond(II)

5d Rubik hypercubes(II)

$$N(6^5)=\begin{cases}\frac{31!}{2}\cdot 60^{31}\cdot\frac{160!}{2}\cdot\frac{12^{160}}{3}\cdot\frac{160!}{2}\cdot\frac{12^{160}}{3}\cdot\frac{320!}{24^{80}}\cdot\frac{6^{320}}{2}\cdot\frac{320!}{24^{80}}\cdot\frac{6^{320}}{2}\cdot\frac{640!}{24^{160}}\cdot\\ \cdot\frac{3^{640}}{3}\cdot\frac{320!}{8!^{40}}\cdot\frac{2^{320}}{2}\cdot\frac{320!}{8!^{40}}\cdot\frac{2^{320}}{2}\cdot\frac{960!}{24!^{40}}\cdot\frac{2^{960}}{2}\cdot\frac{960!}{24!^{40}}\cdot\frac{2^{960}}{2}\cdot\frac{640!}{64!^{10}}\cdot\frac{960!}{96!^{10}}\cdot\\ \frac{640!}{64!^{10}}\cdot\frac{160!}{16!^{10}}\cdot\frac{160!}{16!^{10}}\sim 10^{11441}\end{cases}$$

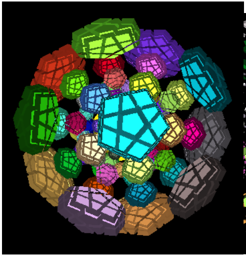
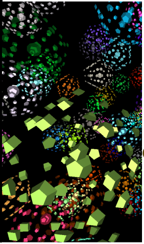
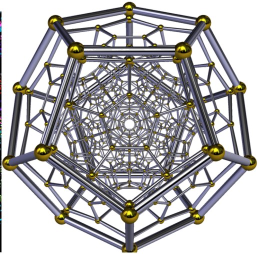
$$N(7^5)=\begin{cases}\frac{32!}{2}\cdot 60^{32}\cdot\frac{80!}{2}\cdot\frac{24^{80}}{2}\cdot\frac{160!}{2}\cdot\frac{12^{160}}{3}\cdot\frac{160!}{2}\cdot\frac{12^{160}}{3}\cdot\frac{80!\cdot 40!}{2}\cdot\frac{6^{80}}{2}\cdot\frac{2^{40}}{2}\cdot\\ \cdot\frac{320!}{24^{80}}\cdot\frac{6^{320}}{2}\cdot\frac{320!}{24^{80}}\cdot\frac{6^{320}}{2}\cdot\frac{320!}{24^{80}}\cdot\frac{6^{320}}{2}\cdot\frac{640!}{24^{160}}\cdot\frac{3^{640}}{3}\cdot\frac{320!}{24^{80}}\cdot\frac{6^{320}}{2}\cdot\frac{240!}{6!^{40}}\cdot\\ \cdot\frac{2^{240}}{2}\cdot\frac{480!}{12!^{40}}\cdot\frac{2^{480}}{2}\cdot\frac{320!}{8!^{40}}\cdot\frac{2^{320}}{2}\cdot\frac{240!}{6!^{40}}\cdot\frac{2^{240}}{2}\cdot\frac{960!}{24!^{40}}\cdot\frac{2^{960}}{2}\cdot\frac{960!}{24!^{40}}\cdot\frac{2^{960}}{2}\cdot\\ \cdot\frac{480!}{12!^{40}}\cdot\frac{2^{480}}{2}\cdot\frac{960!}{24!^{40}}\cdot\frac{2^{960}}{2}\cdot\frac{320!}{8!^{40}}\cdot\frac{2^{320}}{2}\cdot\frac{80!}{8!^{10}}\cdot\frac{240!}{24!^{10}}\cdot\frac{320!}{32!^{10}}\cdot\frac{160!}{16!^{10}}\cdot\frac{80!}{8!^{10}}\cdot\\ \cdot\frac{480!}{48!^{10}}\cdot\frac{960!}{96!^{10}}\cdot\frac{640!}{64!^{10}}\cdot\frac{240!}{24!^{10}}\cdot\frac{960!}{96!^{10}}\cdot\frac{960!}{96!^{10}}\cdot\frac{320!}{32!^{10}}\cdot\frac{640!}{64!^{10}}\cdot\\ \cdot\frac{160!}{16!^{10}}\sim 10^{21503}\end{cases}$$

Higher dimensional Rubik cubes(V): 120-cell

120-cell Rubik puzzle upper bound

Possible combinations (upper bound, unproved):

$$N \leq \frac{600!}{2} \cdot \frac{1200!}{2} \cdot \frac{720!}{2} \cdot \frac{2^{720}}{2} \cdot \frac{6^{1200}}{2} \cdot \frac{12^{600}}{3} \sim 10^{8126} \qquad (11)$$

Monster numbers in group theory/math

Groups are mathematical structures allowing to multiply stuff. Order (number of elements) of the 26 finite simple sporadic groups:

- Mathieu groups: $M_{11} = 7920$, $M_{12} = 95040$ $M_{22} = 443520$, $M_{23} = 10200960$ $M_{24} = 244823040$.
- Janko groups: $J_1 = 175560$, $J_2 = 604800$, $J_3 = 50232960$, $J_4 = 86775571046077562880$.
- Conway groups: $Co(3) = 495766656000$, $Co(2) = 42305421312000$, $Co(1) = 4157776806543360000$.
- Fischer groups: $Fi(22) = 64561751654400$, $Fi(23) = 4089470473293004800$, $Fi(24') = 1255205709190661721292800$.
- Higman–Sims group: $HS = 44352000$.
- McLaughlin group: $McL = 898128000$.
- Held group: $He = 4030387200$.
- Rudvalis group: $Ru = 145926144000$.
- Suzuki sporadic group: $Suz = 448345497600$.
- O'Nan group: $4O'N = 460815505920$.

Monster numbers in group theory/math(II)

- Harada-Norton group: $HN = 273030912000000$.
- Lyons group: $Ly = 51765179004000000$.
- Thompson group: $Th = 90745943887872000$.
- Baby Monster group:
  $B = BM = 4154781481226426191177580544000000$.
- Monster group:
  $M = 2^{46} \cdot 3^{20} \cdot 5^9 \cdot 7^6 \cdot 11^2 \cdot 13^3 \cdot 17 \cdot 19 \cdot 23 \cdot 29 \cdot 31 \cdot 41 \cdot 47 \cdot 59 \cdot 71 =$

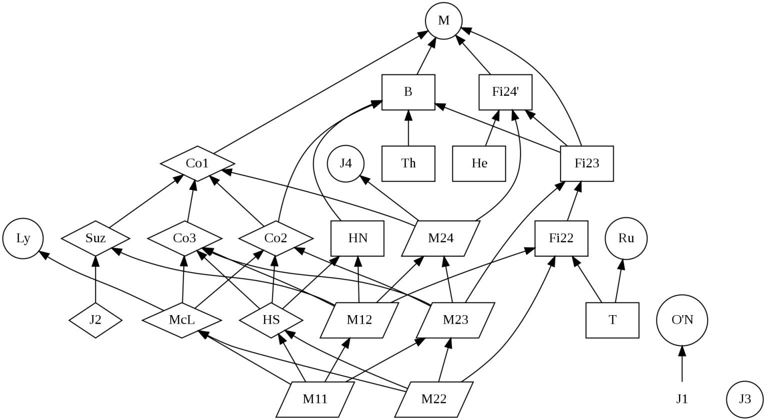
$=808017424794512875886459904961710757005754368000000000$. The Monster group is notable. It has an exact number of elements given by

$$808017424794512875886459904961710757005754368000000000 \approx 8{,}1 \cdot 10^{53}$$

Its lowest dimensional (non trivial) irreducible dimension is 196883d. It is related to the Leech lattice and to the identity $1^2 + 2^2 + \cdots + 24^2 = 70^2$ & to the modular form $J(\tau) = \sum_{m=-1}^{\infty} c_m q^m$ ($\left(\sum_{m=1}^{24} c_m^2\right) \bmod 70 \equiv 42$):

$$J(\tau) = \frac{1}{q} + 196884q + 21493760q^2 + 864299970q^3 + 20245856256q^4 + \cdots$$

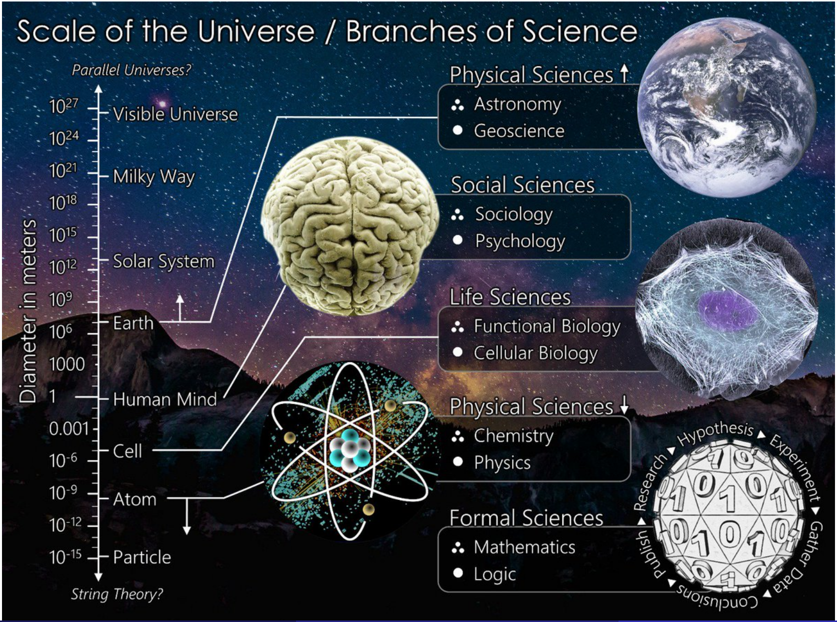
Monster numbers: the Happy Family and the Pariah



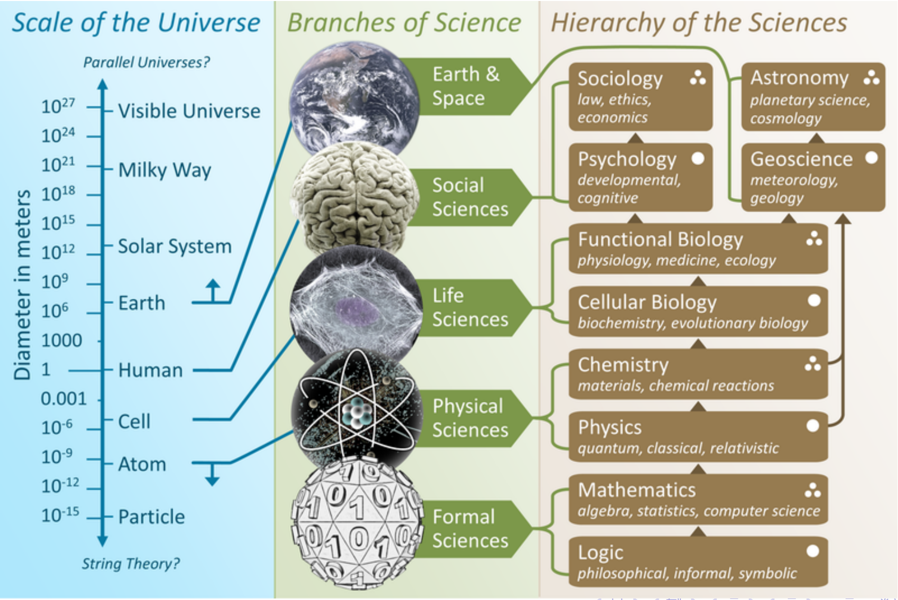
The Pariah are: Ly, O'N, Ru, Janko's J(4), J(3), J(1). The Others are the Happy Family. Both sets are inside the Friendly Giant (the Monster group).

Contenido

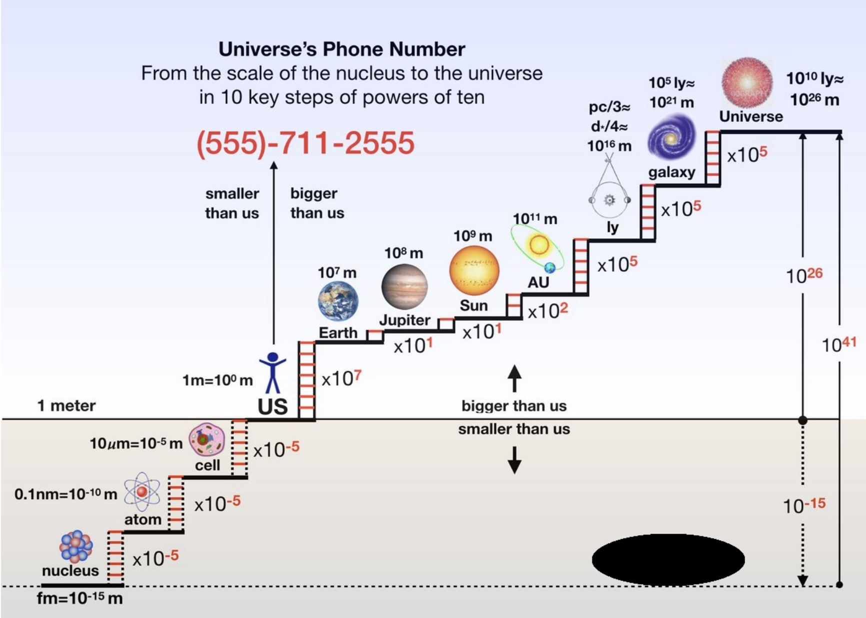
Scales/Escalas (I)



Scales/Escalas (II)

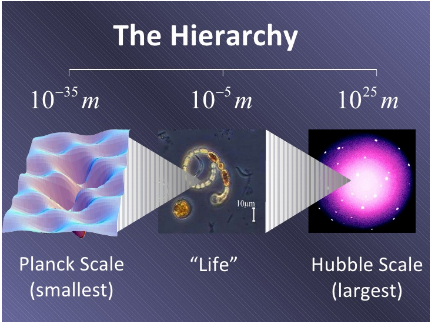


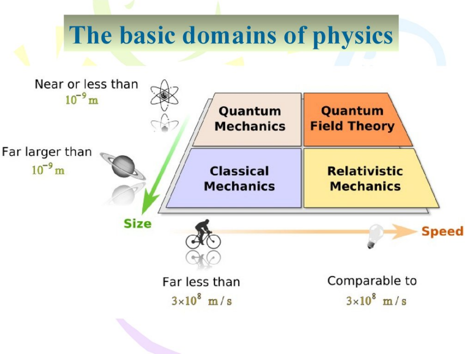
Scales/Escalas (III): The Phone Number of the Universe



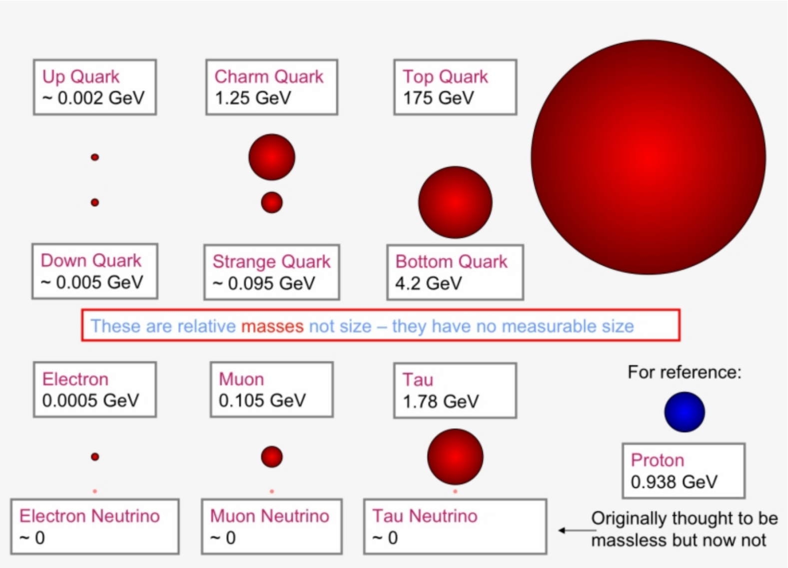
Scales/Escalas (IV): hierarchy and domains of physics/chemistry

Jerarquías y dominios de la Física conocida:

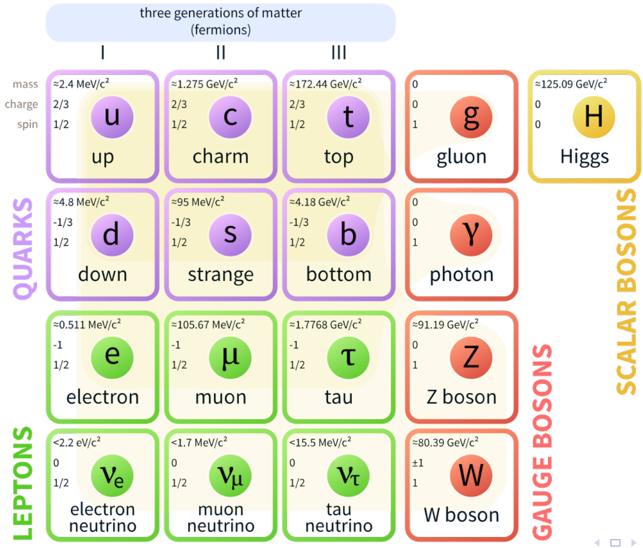




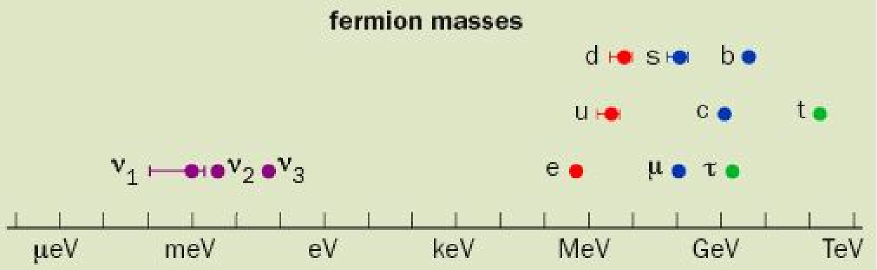
Scales/Escalas (V): particle mass scales in the SM



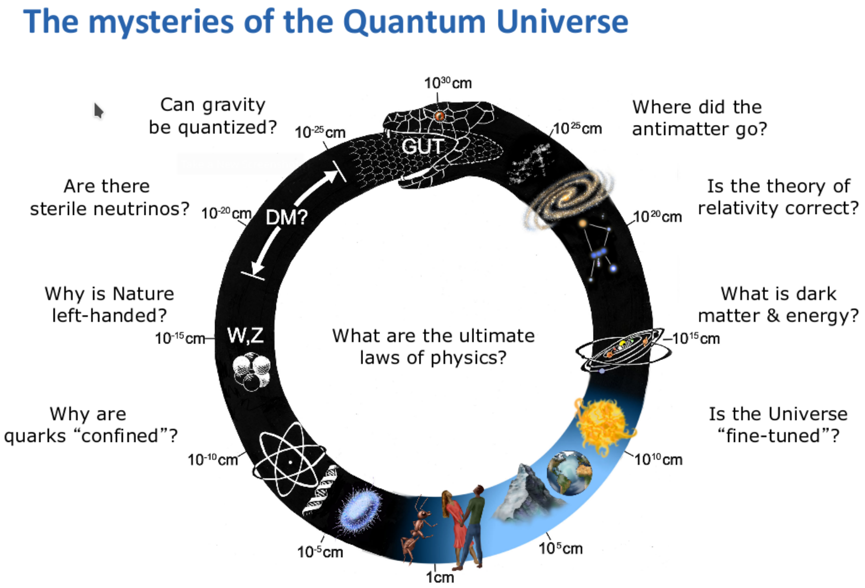
Standard Model of Elementary Particles



Scales/Escalas (V): particle mass scales in the SM(III)



Higgs mass: 125 GeV, W boson mass is about 80 GeV, Z boson about 91 GeV. Thus weak gauge bosons and the Higgs are about 100 GeV=0.1 TeV. Why 100 GeV? Nobody knows!!!!


The mysteries of the Quantum Universe
Can gravity be quantized?
Are there sterile neutrinos?
Why is Nature left-handed?
Why are quarks "confined"?
Where did the antimatter go?
Is the theory of relativity correct?
What is dark matter & energy?
Is the Universe "fine-tuned"?
What are the ultimate laws of physics?
10^{30}cm
10^{-25}cm
10^{25}cm
10^{-20}cm
10^{20}cm
10^{-15}cm
10^{15}cm
10^{-10}cm
10^{10}cm
10^{-5}cm
10^{5}cm
1cm
GUT
DM?
W,Z

Length scales/Escalas de longitud

- The observable Universe (it could be really bigger than that) has a radius about $R_U = 10^{26} m \sim 100 Ym$.
- The shortest possible length is the so-called Planck length:

Planck length

$$L_p = \sqrt{\frac{G_N \hbar}{c^3}} \simeq 1{,}6 \cdot 10^{-35} m \sim 10^{-33} cm$$

- Intermediate microsscales: bacteria/viruses ($R_V \sim 10^{-6} m$), atoms $R_a \sim 10^{-10} m$, nuclei $R_n \sim 10^{-15} m$, subatomic particles $R_s \leq 10^{-18} m$.
- Intermediate macroscales: humans and earthling macrolife $10^{-2} m \leq R \leq 10 m$, $R_{\oplus} \simeq 6{,}4 \cdot 10^6 m$, $R_{\odot} \simeq 7 \cdot 10^8 m$, $d(\oplus\odot) \simeq 1{,}5 \cdot 10^{11} m$, $1 lyr \simeq 9{,}45 \cdot 10^{15} m$, $D_{MW} = 10^5 lyrs \simeq 10^{21} m$, $D_U \sim 10^6 D_{MW}$.

Why is the Universe big? (I)

Why $R_{Earth} >> R_{atom}$? Answer: It is related to the relative force between gravity and electromagnetism due to a Gauss law! In 3d space (4d spacetime) both, Coulomb and Newton laws are inversely proportional to the square of distance (spherical shells!). Then:

$$\frac{F_E}{F_a} = \frac{R_E^2}{R_a^2}$$

and from this you get

$$\frac{R_E}{R_a} = \sqrt{\frac{Ke^2}{GM_p^2}} = \left(\frac{e}{M_p}\right)\left(\sqrt{\frac{K_C}{G_N}}\right) \sim 10^{18}$$

The reason is that the ratio of the electron to proton mass, and the square root of Coulomb to Newton constant is big. Earth is big, because of the nature of proton mass to electron charge, and the ratio of the coulomb to newton constant. Thus, you can not find atomic planets or atoms with the size of the Earth...Not go crazy and ask to change of these values...

Why $R_{Elephant} >> R_{atom}$?
Answer:Similarly, you can can compute the ratio between the the gravitational energy of an elephan and the electric energy of any atom:

$$\frac{E(g)}{V_{El}} = \frac{E(atom)}{V_{atom}}$$

Then, you get

$$\frac{R_{El}}{R_{atom}} = \left(\sqrt{\frac{Ke^2}{GM_p^2}}\right)^{-2/3} = \left(\frac{GM_p^2}{K_C e^2}\right)^{1/3} \sim 10^2(10)$$

Therefore, elephants or humans are constrained to be a few meters height...Great!

Why is the Universe big? (III)

Why $R_{atom} = 10^5 R_{nucleus}$? Answer: It shows to be the hardest to understand. The secret behind this answer lies in the Yukawa force and the exponential screening (short-distance) behaviour of nuclear forces that make them to be confined to a few proton radii (and of course, to the coupling constants). Take for instance the strong force case, then

$$g_s^2 \frac{\exp(-r/r_0)}{r^2} = \frac{K_C e^2}{r^2}$$

then

$$\frac{r_{nucleus}}{r_{atom}} = \sqrt{\frac{\alpha_s}{\alpha}} \exp(-r/2r_0)$$

Plugging $\alpha_s \sim 1$, $\alpha_0 = 1/137$, $r/2r_0 \sim 5$, you guess that the above ratio is

$$\frac{r_{nucleus}}{r_{atom}} = 10^{-5}$$

Fantastic! Allons-y! Spoiler: Proton decay is expected naturally at some point between 10^{45}yrs or 10^{120} yrs from "standard assumptions" of virtual black holes or space-time foam expectations.

Biggest and smallest stars

Stellar HR diagrama: OBAFGKM. SubMain sequence types: LTY. WD types: D+(ABCOZQX)+(PHEV).

- Compact stars: $D(WD) \sim 10^4 km = 10^7 m$, $D(NS) \sim 20km = 2 \cdot 10^4 m$, $D(QS) = 10km = 10^4 m$, $D(PS) = 10cm = 0{,}1m$, $D(BH_S) = 2{,}95km(M/M_\odot)$, BH singularities/point particles $D = 0m$.
- Typical star diameter: $D_\odot = 6{,}96 \cdot 10^8 m = 696000km$.
- M dwarf star diameter: $D_\star \approx 0{,}1D_\odot$.
- O type supergiant Pistol Star (blue hypergiant): $D_\star = 306D_\odot$.
- Supergiant stars (examples): biggest star known (circa 2020) is Stephenson 2-18 (St2-18), since $R = 2150R_\odot$ (red supergiant), Betelgeuse $D_B = 887D_\odot$.
- Brown dwarf sizes: $M > 13M_J$ up to $M = 0{,}075M_\odot$ and $0{,}1R_\odot < R < 1{,}5R_\odot$, approximately.

Scales of cosmic time(I)

Planck time(Shortest possible time is the Planck time)

$$t_P = \frac{L_p}{c} = \sqrt{\frac{G_N \hbar}{c^5}} \sim 10^{-43} s = 10^{-19} ys \quad (12)$$

- Inflation era was about $10^{-35}s$ after the $t = 0s$.
- Quark-gluon-plasma era was about $10^{-10}s$ the creation time.
- First atoms formed about $t \simeq 380000yrs \sim 10^{13}s$ after $t = 0s$.
- First stars and galaxies were forming after millions and billions of years. Cosmic current age is about $t_u \approx 14Gyr = 1{,}4 \cdot 10^{10}yrs \sim 10^{18}s$.
- Sun-like stars live about $10^9 - 10^{10}yrs$. Red dwarf stars can survive during $10^{12} - 10^{13}yrs$. Then they become blue dwarf stars during a few $10^9 yrs$ and finally become black dwarf stars. Similarly, white dwarf stars (WD) can also life trillions $\sim 10^{12}yrs$ before black dwarf phase.
- Proton lifetime is about $t(p^+) \geq 10^{34}yrs = 10^{41}s$. Proton decay is expected naturally at some point between 10^{45}yrs or 10^{122}yrs.

Scales of cosmic time(II)

- Solar mass non-rotating black holes have a lifetime about $10^{67} yrs \sim 10^{74} s$.
- The most massive black dwarf stars are expected to explode into black supernovae in about $10^{1100} yrs - 10^{32000} yrs$ (supposing proton are stable enough to have survived till then).
- General prediction of GUTs and TOEs: proton could decay in time

$$t_d \simeq \frac{M_X^4}{\alpha_X^2 M_p^5}$$

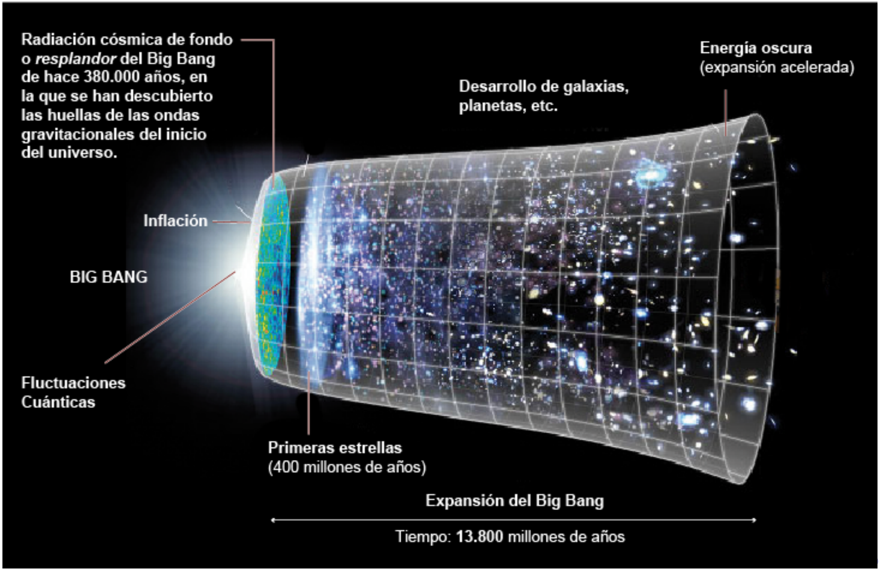
Virtual black holes can trigger proton decays in a time

$$t_d(VBH) = \frac{M_P^4}{m_p^5} \simeq 10^{45} yrs$$

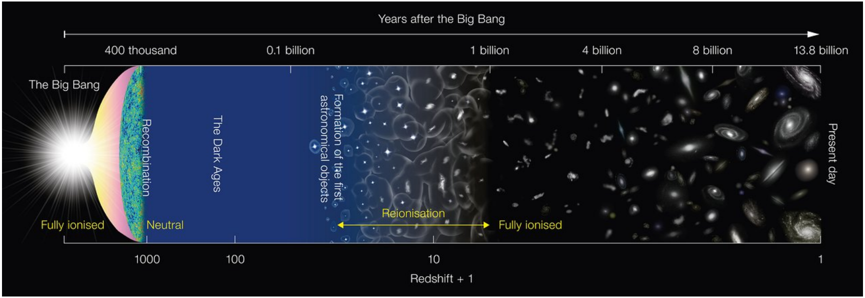
- Other virtual quantum black hole/spacetime foam predictions provide $t_p \simeq (M_P/m_p)^8/m_p \sim 10^{120} yrs$. Other: $t(e^-, Borexino) > 10^{29} yrs$.

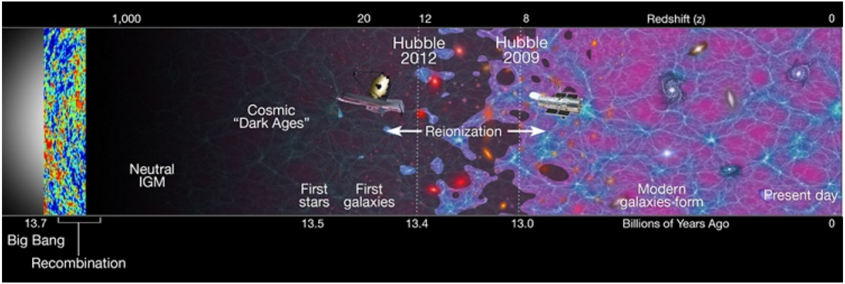
Scales of cosmic time(III)

EVOLUCIÓN DEL COSMOS

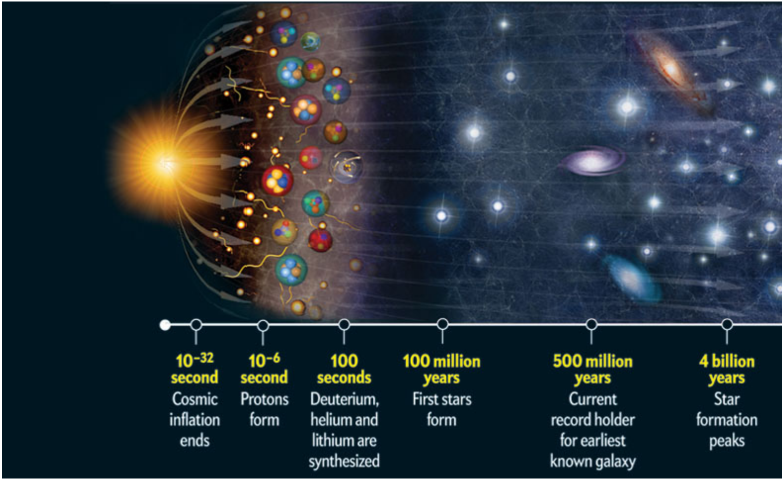


Scales of cosmic time(IV)





Scales of cosmic time(V)



Others: some unstable particles/nuclei have very tiny life-times. Higgs boson half-life is 0,16zs, top quark 0,5ys, ^{7}H 23ys. In the other side: Te-128 half-life is $2{,}2 \cdot 10^{24}$ yrs, i.e., 2.2Yyrs.

Scales of mass(I)

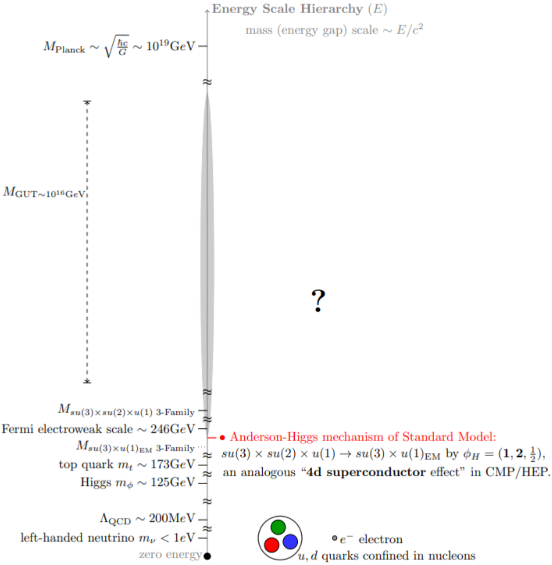
- Universe mass is about: $10^{53}kg$.
- Supermassive/Ultramassive black holes: $10^9 - 10^{10}M_\odot \sim 10^{39}k$.
- Galaxies(normal matter): $M_G(Baryon) \sim 10^5 \cdot 10^6 M_\odot \sim 10^{41}kg$. Real mass of galaxies seems to be $10M_G(Baryon) \sim 10^{42}kg$. Dark matter is supposed to be the up to 90 % of the mass or more.
- The most massive stars, Pop-III, were about $10^2 - 10^3 M_\odot$.
- The current least massive stars are about 0.1 solar masses, i.e., $10^{29}kg$. Neutron stars and WD stars have limites about 3 and 1.44 solar masses.
- Brown dwarfs: $10^{28} - 10^{29}kg$.
- Macrolife: $10 - 10^2 kg$. Microlife: $m \sim \mu g$. Planck mass (minmax. mass): $m_P = \sqrt{c\hbar/G} \sim 22\mu g = 2{,}2 \cdot 10^{-8}kg$.
- Proton: 1GeV ($\sim 10^{-27}kg$), electron $\sim 10^{-30}kg$, neutrino mass $m_\nu < 10^{-37}kg$, energía oscura de Garidi $m_{de} \sim 10^{-69}kg$ ($M_U/M_{de} \sim 10^{122}$).

Scales of energy

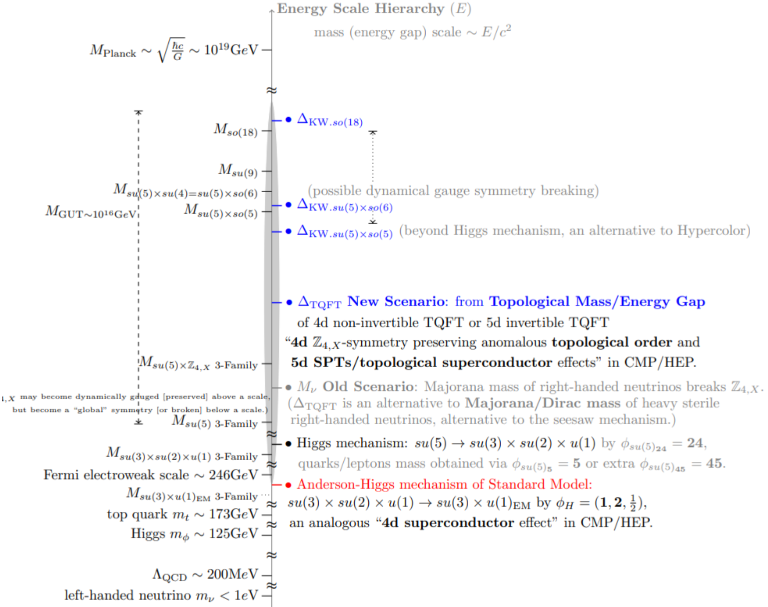
Typical unit, the electron-volt: $1eV \sim 10^{-19} J$.

- Planck energy: $E_p = \sqrt{\frac{\hbar c^5}{G}} \sim 10^{16} TeV = 10^{19} GeV$. TOE a priori scale.
- Higgs mass: 125 GeV, top quark 173 GeV.
- 140 MeV: confinement scale.
- eV scale: atomic levels.
- meV, μeV scale: neutrino mass scale.
- Dark energy scale: $10^{-33} eV$.
- Dark matter energy scale mystery: between 80 orders of magnitude.

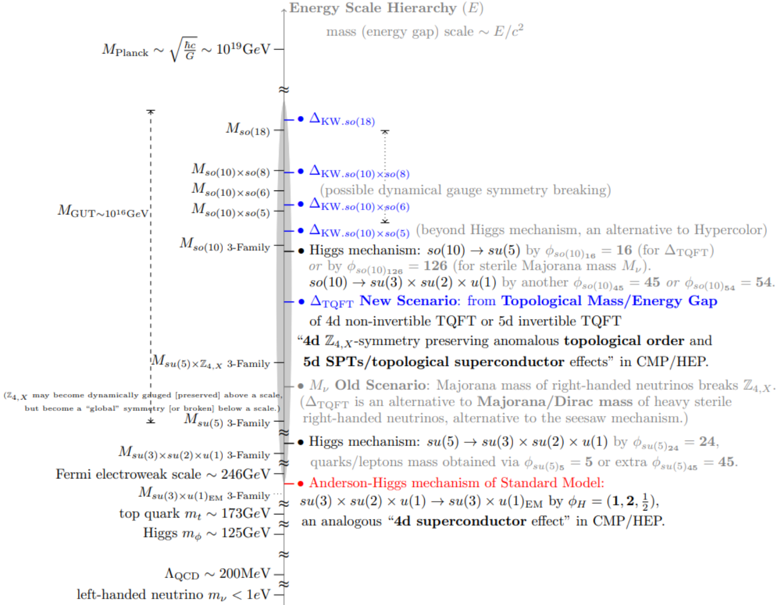
Scales of energy(II)



Scales of energy(II)



Scales of energy(II)



Scales of temperature

Planck temperature: absolute hot, spacetime melting point

$$T_P = \sqrt{\frac{\hbar c^5}{G k_B}} \sim 10^{32} K$$

- Hagedorn stringy temperature: $T_H \sim 10^{30} - 10^{32} K$.
- QGP (quark-gluon plasma): $T(QGP) \sim 10^{12} K$. Nuclear pasta (gnocchi, spaghetti, lasagna, bucatini -antispaghetti, Swiss cheese).
- Plasma (electron): $T_e \sim 10^6 K$.
- Sun: $T_\odot(sup) \sim 6000K$ $(T_\odot(int) \sim 10^6 K)$.
- Hottest stars: $T \sim 10^6 K$(sup).
- Earth temperature: $T_E \sim 3 \cdot 10^2 K$.
- CMB: $T_\gamma = 2{,}73K$, $T_\nu = 1{,}945K$, $T_G = 0{,}9K$.
- Bose-Einstein condensates: $T \sim 0 \quad K$.
- Absolute zero: $T = 0 \quad K$(impossible due to vacuum fluct. and/or 3rd law of thermodynamics).

Scales of density

- Vacuum density: $10^{-27} kg/m^3$.
- Air: $1{,}3 kg/m^3$. Aerogel: $2kg/m^3$.
- Water: $1000 kg/m^3$.
- Earth average density: $5000 kg/m^3$.
- Diamond: $3{,}5 kg/m^3$.
- Metals: $(5-20)\cdot 10^3 kg/m^3$. Osmium: $22610 kg/m^3$.
- Sun density: $M_\odot/V_\odot = \rho_\odot = 10^{30}/(4\cdot 7^3\cdot 10^{24}) \sim 1{,}4\cdot 10^3 kg/m^3$. Sun core density: $1{,}5\cdot 10^5 kg/m^3$.
- White dwarf stars: $10^9 kg/m^3$.
- Neutron stars: $10^{16}-10^{18} kg/m^3$. Atomic/nuclei density: $10^{17} kg/m^3$.
- Preon stars (hypothesis): $10^{30} kg/m^3$.
- Planck stars (hypothesis): $10^{96} kg/m^3$. Black holes: $\rho \propto 1/M^2$. Earth mass BH: $10^{30} kg/m^3$. M_9 BH: $\rho < \rho_w$. Micro BH: $\rho >> \rho_P$!(Electron BH: $\rho \sim 10^{140} kg/m^3$!!!!
- Planck density (maximal): $10^{97} kg/m^3$. Spacetime singularity/spacetime point: $\rho = \infty$!(likely unphysical)

Scales of velocity

- Gauge boson (long-range) masses (Special relativity (single time) limit: $v \leq c$): $m_\gamma = m_g = m_G = 0 \rightarrow v_\gamma = v_g = v_G = c$.
- Fastest massive particles (cosmic rays, neutrinos): $m \sim c$.
- Fastest relativistic stars/black holes/NS: $v = (0{,}01 - 0{,}1)c$.
- Escape velocities: $v(EH) = c = 299792458 m/s$ (BH def.).
- Orbital velocities: $v_{orb} \sim (1 - 10^3) km/s = 10^6 m/s = (0{,}00001c - 0{,}01c)/3$.
- Cosmic speeds: 1st and 2nd cosmic speeds are about 10km/s, 3rd (solar system escape velocity) is about 42km/s, 4th (Milky Way escape velocity) is about 350km/s, 5th is the escape velocity from this Universe $v_5 = \sqrt{\dfrac{2GM_U}{R_U}}$.

Scales of velocity(II)

Other Earth-Sun velocities:

- Rotational speed of Earth on the equator is about 1670km/h or 0.46km/s.
- Rotational speed of Earth around the sun is about 107000km/h, or about 30 km/s.
- Rotational speed of the solar system (the sun) around the Milky Way is about 220km/s (828000km/h).
- The Milky Way spins at about 270 km/s with respect to its center. Milky Way speed towards the Big Attractor is about 611km/s, or about 2.2 million km/s. Milky Way speed with respect to the CMB is about 2268000km/h or about 630 km/s.
- Sound speed (air, Earth): $333 m/s \sim 3 \cdot 10^2 m/s = c/10^6$.
- Human speeds: $(1-10000) m/s$.
- Slowest speeds (quantum): $v \sim 0 m/s$ ($v = 0$m/s impossible due to quantum vacuum effects).

Scales of acceleration/gravitational fields

- Planck maximal acceleration: $a_P = \sqrt{c^7/G_N\hbar} \approx 5{,}56 \cdot 10^{51} m/s^2$. $a_p \simeq 5{,}67 \cdot 10^{50} g$. [Planck maximal force $c^4/G_N \simeq 1{,}2103 \cdot 10^{44} N$.]
- Earth gravitational field in its surface: $g \simeq 9{,}81 m/s^2 \sim 10 m/s^2$.
- Solar mass neutron star surface gravity: $g(s, R = 10km) = 1{,}3 \cdot 10^6 m/s^2 \sim 10^5 g$. Solar mass preonic star: $g(s, R = 1cm) = 1{,}3 \cdot 10^{24} m/s^2 \sim 10^{23} g$.
- BH singularity: $g = \infty m/s^2$ (likely unphysical!).
- Solar mass black hole ($M_{BH} = 3M_\odot$): $\kappa = g(s) = 5{,}1 \cdot 10^{12} m/s^2 \sim 5{,}1 \cdot 10^{11} g$.
- Electron mass black hole (likely impossible!): $g_s = 3{,}33 \cdot 10^{73} m/s^2 \sim 3 \cdot 10^{72} g$.
- Schwinger critical gravitational field for the electron: $g_S = mc^3/\hbar \simeq 3{,}7 \cdot 10^{28} m/s^2 \sim 4 \cdot 10^{27} g$
- Lowest acceleration? MONDian hypothesis: $a_0 \sim 10^{-10} m/s^2$. $a = 0 m/s^2$ likely impossible due to quantum effects!

Planck scale limits and units

Table 1: Dimensional universal physical constants normalized with Planck units			
Constant	**Symbol**	**Dimension in SI Quantities**	**Value (SI units)**
Speed of light in vacuum	c	$L\,T^{-1}$	$299\,792\,458$ m·s^{-1}[2] *(exact by definition)*
Gravitational constant	G	$L^3\,M^{-1}\,T^{-2}$	$6.674\,30(15) \times 10^{-11}$ m^3·kg^{-1}·s^{-2}[3]
Reduced Planck constant	$\hbar = \frac{h}{2\pi}$ where h is the Planck constant	$L^2\,M\,T^{-1}$	$1.054\,571\,817... \times 10^{-34}$ J·s[4] *(defined as* $\frac{6.626\,070\,15 \times 10^{-34}\ J{\cdot}s}{2\pi}$ *exactly)*
Boltzmann constant	k_B	$L^2\,M\,T^{-2}\,\Theta^{-1}$	$1.380\,649 \times 10^{-23}$ J·K^{-1}[5] *(exact by definition)*
Coulomb constant	$k_e = \frac{1}{4\pi\varepsilon_0}$ where ε_0 is the permittivity of free space	$L^3\,M\,T^{-2}\,Q^{-2}$	$8.987\,551\,7923(14) \times 10^9$ kg·m^3·s^{-4}·A^{-2}[6]

Key: L = length, M = mass, T = time, Q = electric charge, Θ = temperature.

Planck scale limits and units

A property of Planck units is that in order to obtain the value of any of the physical constants above it is enough to replace the dimensions of the constant with the corresponding Planck units. For example, the gravitational constant (G) has as dimensions $L^3 M^{-1} T^{-2}$. By replacing each dimension with the value of each corresponding Planck unit one obtains the value of $(1\ l_P)^3 \times (1\ m_P)^{-1} \times (1\ t_P)^{-2}$ = $(1.616255\times10^{-35}\ \text{m})^3 \times (2.176435\times10^{-8}\ \text{kg})^{-1} \times (5.391247\times10^{-44}\ \text{s})^{-2} = 6.674\ldots\times10^{-11}\ \text{m}^3\ \text{kg}^{-1}\ \text{s}^{-2}$ (which is the value of G).

This is the consequence of the fact that the system is internally coherent. For example, the gravitational attractive force of two bodies of 1 Planck mass each, set apart by 1 Planck length is 1 coherent Planck unit of force. Likewise, the distance traveled by light during 1 Planck time is 1 Planck length.

To determine, in terms of SI or another existing system of units, the quantitative values of the five base Planck units, those two equations and three others must be satisfied:

$$l_P = c\, t_P$$

$$F_P = \frac{m_P l_P}{t_P^2} = G\,\frac{m_P^2}{l_P^2}$$

$$E_P = \frac{m_P l_P^2}{t_P^2} = \hbar\,\frac{1}{t_P}$$

$$E_P = \frac{m_P l_P^2}{t_P^2} = k_B\, T_P\,.$$

$$F_P = \frac{m_P l_P}{t_P^2} = \frac{1}{4\pi\varepsilon_0}\,\frac{q_P^2}{l_P^2}$$

Planck scale limits and units

Table 2: Base Planck units			
Name	**Dimension**	**Expression**	**Value (SI units)**
Planck length	Length (L)	$l_P = \sqrt{\frac{\hbar G}{c^3}}$	1.616 255(18) × 10^{-35} m[7]
Planck mass	Mass (M)	$m_P = \sqrt{\frac{\hbar c}{G}}$	2.176 435(24) × 10^{-8} kg[8]
Planck time	Time (T)	$t_P = \sqrt{\frac{\hbar G}{c^5}}$	5.391 247(60) × 10^{-44} s[9]
Planck temperature	Temperature (Θ)	$T_P = \sqrt{\frac{\hbar c^5}{G k_B^2}}$	1.416 785(16) × 10^{32} K[10]
Planck charge	Electric charge (Q)	$q_P = \sqrt{\frac{\hbar c}{k_e}} = \sqrt{4\pi\varepsilon_0 \hbar c} = \sqrt{\frac{4\pi\hbar}{\mu_0 c}} = \frac{e}{\sqrt{\alpha}}$	1.875 545 956(41) × 10^{-18} C[11][4][2]

Planck scale limits and units

Table 3: Coherent derived units of Planck units		
Derived unit of	Expression	Approximate SI equivalent
area (L^2)	$l_P^2 = \frac{\hbar G}{c^3}$	2.6121×10^{-70} m²
volume (L^3)	$l_P^3 = \left(\frac{\hbar G}{c^3}\right)^{\frac{3}{2}} = \sqrt{\frac{(\hbar G)^3}{c^9}}$	4.2217×10^{-105} m³
momentum (LMT^{-1})	$m_P c = \frac{\hbar}{l_P} = \sqrt{\frac{\hbar c^3}{G}}$	6.5249 kg·m/s
energy (L^2MT^{-2})	$E_P = m_P c^2 = \frac{\hbar}{t_P} = \sqrt{\frac{\hbar c^5}{G}}$	1.9561×10^{9} J
force (LMT^{-2})	$F_P = \frac{E_P}{l_P} = \frac{\hbar}{l_P t_P} = \frac{c^4}{G}$	1.2103×10^{44} N
density ($L^{-3}M$)	$\rho_P = \frac{m_P}{l_P^3} = \frac{\hbar t_P}{l_P^5} = \frac{c^5}{\hbar G^2}$	5.1550×10^{96} kg/m³
acceleration (LT^{-2})	$a_P = \frac{c}{t_P} = \sqrt{\frac{c^7}{\hbar G}}$	5.5608×10^{51} m/s²
frequency (T^{-1})	$f_P = \frac{c}{l_P} = \sqrt{\frac{c^5}{\hbar G}}$	1.8549×10^{43} Hz

Planck scale limits and units

Table 4: Original Planck units				
Name	**Dimension**	**Expression**	**Value in SI units**	**Value in modern Planck units**
Original Planck length	Length (L)	$\sqrt{\frac{hG}{c^3}}$	$4.051\,35 \times 10^{-35}$ m	$\sqrt{2\pi} \times l_P$
Original Planck mass	Mass (M)	$\sqrt{\frac{hc}{G}}$	$5.455\,51 \times 10^{-8}$ kg	$\sqrt{2\pi} \times m_P$
Original Planck time	Time (T)	$\sqrt{\frac{hG}{c^5}}$	$1.351\,38 \times 10^{-43}$ s	$\sqrt{2\pi} \times t_P$
Original Planck temperature	Temperature (Θ)	$\sqrt{\frac{hc^5}{Gk_B^2}}$	$3.551\,35 \times 10^{32}$ K	$\sqrt{2\pi} \times T_P$

Planck scale limits and units

Table 6: Today's universe in Planck units.		
Property of present-day observable universe	**Approximate number of Planck units**	**Equivalents**
Age	$8.08 \times 10^{60}\ t_P$	4.35×10^{17} s, or 13.8×10^{9} years
Diameter	$5.4 \times 10^{61}\ l_P$	8.7×10^{26} m or 9.2×10^{10} light-years
Mass	approx. $10^{60}\ m_P$	3×10^{52} kg or 1.5×10^{22} solar masses (only counting stars) 10^{80} protons (sometimes known as the Eddington number)
Density	$1.8 \times 10^{-123}\ \rho_P$	9.9×10^{-27} kg m^{-3}
Temperature	$1.9 \times 10^{-32}\ T_P$	2.725 K temperature of the cosmic microwave background radiation
Cosmological constant	$5.6 \times 10^{-122}\ t_P^{-2}$	1.9×10^{-35} s^{-2}
Hubble constant	$1.18 \times 10^{-61}\ t_P^{-1}$	2.2×10^{-18} s^{-1} or 67.8 (km/s)/Mpc

Planck scale limits and units

Table 7: How Planck units simplify the key equations of physics		
	SI form	Planck units form
Newton's law of universal gravitation	$F = G\frac{m_1 m_2}{r^2}$	$F = \frac{m_1 m_2}{r^2}$
Einstein field equations in general relativity	$G_{\mu\nu} = 8\pi \frac{G}{c^4} T_{\mu\nu}$	$G_{\mu\nu} = 8\pi T_{\mu\nu}$
Mass–energy equivalence in special relativity	$E = mc^2$	$E = m$
Energy–momentum relation	$E^2 = m^2 c^4 + p^2 c^2$	$E^2 = m^2 + p^2$
Thermal energy per particle per degree of freedom	$E = \frac{1}{2} k_B T$	$E = \frac{1}{2} T$
Boltzmann's entropy formula	$S = k_B \ln \Omega$	$S = \ln \Omega$
Planck–Einstein relation for energy and angular frequency	$E = \hbar\omega$	$E = \omega$
Planck's law (surface intensity per unit solid angle per unit angular frequency) for black body at temperature T.	$I(\omega, T) = \frac{\hbar\omega^3}{4\pi^3 c^2} \frac{1}{e^{\frac{\hbar\omega}{k_B T}} - 1}$	$I(\omega, T) = \frac{\omega^3}{4\pi^3} \frac{1}{e^{\omega/T} - 1}$
Stefan–Boltzmann constant σ defined	$\sigma = \frac{\pi^2 k_B^4}{60\hbar^3 c^2}$	$\sigma = \frac{\pi^2}{60}$
Bekenstein–Hawking black hole entropy[50]	$S_{BH} = \frac{A_{BH} k_B c^3}{4G\hbar} = \frac{4\pi G k_B m_{BH}^2}{\hbar c}$	$S_{BH} = \frac{A_{BH}}{4} = 4\pi m_{BH}^2$
Schrödinger's equation	$-\frac{\hbar^2}{2m}\nabla^2\psi(\mathbf{r},t) + V(\mathbf{r},t)\psi(\mathbf{r},t) = i\hbar\frac{\partial\psi(\mathbf{r},t)}{\partial t}$	$-\frac{1}{2m}\nabla^2\psi(\mathbf{r},t) + V(\mathbf{r},t)\psi(\mathbf{r},t) = i\frac{\partial\psi(\mathbf{r},t)}{\partial t}$
Hamiltonian form of Schrödinger's equation	$H\lvert\psi_t\rangle = i\hbar\frac{\partial}{\partial t}\lvert\psi_t\rangle$	$H\lvert\psi_t\rangle = i\frac{\partial}{\partial t}\lvert\psi_t\rangle$
Covariant form of the Dirac equation	$(i\hbar\gamma^\mu\partial_\mu - mc)\psi = 0$	$(i\gamma^\mu\partial_\mu - m)\psi = 0$
Unruh temperature	$T = \frac{\hbar a}{2\pi c k_B}$	$T = \frac{a}{2\pi}$
Coulomb's law	$F = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}$	$F = \frac{q_1 q_2}{r^2}$
Maxwell's equations	$\nabla\cdot\mathbf{E} = \frac{1}{\epsilon_0}\rho$ $\nabla\cdot\mathbf{B} = 0$ $\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}$ $\nabla\times\mathbf{B} = \frac{1}{c^2}\left(\frac{1}{\epsilon_0}\mathbf{J} + \frac{\partial\mathbf{E}}{\partial t}\right)$	$\nabla\cdot\mathbf{E} = 4\pi\rho$ $\nabla\cdot\mathbf{B} = 0$ $\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}$ $\nabla\times\mathbf{B} = 4\pi\mathbf{J} + \frac{\partial\mathbf{E}}{\partial t}$
Ideal gas law	$PV = nRT$	$PV = NT$

Planck scale limits and units

Table 8: Equivalences between Planck base units[51]

	Planck length (l_P)	Planck mass (m_P)	Planck time (t_P)	Planck temperature (T_P)	Planck charge (q_P)
Planck length (l_P)	—	$l_P = \frac{\hbar}{m_P c}$	$l_P = t_P c$	$l_P = \frac{\hbar c}{T_P k_B}$	$l_P = \frac{\hbar}{q_P c}\sqrt{\frac{G}{k_e}}$
Planck mass (m_P)	$m_P = \frac{\hbar}{l_P c}$	—	$m_P = \frac{\hbar}{t_P c^2}$	$m_P = \frac{T_P k_B}{c^2}$	$m_P = q_P\sqrt{\frac{k_e}{G}}$
Planck time (t_P)	$t_P = \frac{l_P}{c}$	$t_P = \frac{\hbar}{m_P c^2}$	—	$t_P = \frac{\hbar}{T_P k_B}$	$t_P = \frac{\hbar}{q_P c^2}\sqrt{\frac{G}{k_e}}$
Planck temperature (T_P)	$T_P = \frac{\hbar c}{l_P k_B}$	$T_P = \frac{m_P c^2}{k_B}$	$T_P = \frac{\hbar}{t_P k_B}$	—	$T_P = \frac{q_P c^2}{k_B}\sqrt{\frac{k_e}{G}}$
Planck charge (q_P)	$q_P = \frac{\hbar}{l_P c}\sqrt{\frac{G}{k_e}}$	$q_P = m_P\sqrt{\frac{G}{k_e}}$	$q_P = \frac{\hbar}{t_P c^2}\sqrt{\frac{G}{k_e}}$	$q_P = \frac{T_P k_B}{c^2}\sqrt{\frac{G}{k_e}}$	—

- Critical Schwinger electric field:

$$E_S = \frac{m^2c^3}{e\hbar} \simeq 1{,}3\cdot 10^{18}\, V/m, \quad E_S^w = \frac{m^2c^4}{K_C e^3} = E_S\alpha^{-1} \simeq 1{,}81\cdot 10^{20}\, V/m$$

- Critical Schwinger magnetic field:

$$B_S = \frac{E_S}{c} = \frac{m^2c^2}{e\hbar} \simeq 4{,}4\cdot 10^{9}\, T, \quad B_S^w = \frac{m^2c^3}{K_C e^3} = B_S\alpha^{-1} \simeq 6{,}05\cdot 10^{11}\, T$$

- Highest B-fields (human-made): $B \sim 10 - 10^2\, T$.
- Highest B-fields (Nature): Earth's $25 < B < 65\mu T$, magnetar $10^9 - 10^{11}\, T$ (fast NS), typical MW-like galaxy $1nT$.
- Weak and Strong nuclear fields are bigger at femtoscale or below.
- Smallest fields are equal to zero or almost: zero point fluctuations likely made non-zero values impossible unless net charge is zero.
- Residual electromagnetic forces provide chemistry and intermolecular forces responsible of the macroworld!
- Residual strong nuclear forces provide nuclear stability.

The Strange Multiverse and Multiverse of Madness

The Strange Multiverse is...strange...Who am I to judge?

Scales of entropy/number of particles

- Considering there Universe is baryonic, we have about $10^{53}/10^{-27} \sim 10^{80}$ protons in the Universe.
- String theory predicts a vast Multiverse or Landscape, with about 10^{500} or 10^{272000} possible vacua compatible with similar Universes.
- Entropy of the Universe: $10^{66} J/K \sim 10^{88} bans$. Note than Shannon's entropy gives you $\Omega = e^{S/k_B}$ microstates.

objects	entropy	energy
10^{22} stars	10^{79}	$\Omega_{\text{stars}} \sim 10^{-3}$
relic neutrinos	10^{88}	$\Omega_{\nu} \sim 10^{-5}$
stellar heated dust	10^{86}	$\Omega_{\text{dust}} \sim 10^{-3}$
CMB photons	10^{88}	$\Omega_{\text{CMB}} \sim 10^{-5}$
relic gravitons	10^{86}	$\Omega_{\text{grav}} \sim 10^{-6}$
stellar BHs	10^{97}	$\Omega_{\text{SBH}} \sim 10^{-5}$
single supermassive BH	10^{91}	$10^{7} M_{\odot}$
$10^{11} \times 10^{7} M_{\odot}$ SMBH	10^{102}	$\Omega_{\text{SMBH}} \sim 10^{-5}$
holographic upper bound	10^{123}	$\Omega = 1$

Component	Entropy S [k]
Cosmic Event Horizon	$2.6 \pm 0.3 \times 10^{122}$
SMBHs	$1.2^{+1.1}_{-0.7} \times 10^{103}$
Stellar BHs (2.5 – 15 $M_{\odot}$)	$2.2 \times 10^{96^{+0.6}_{-1.2}}$
Photons	$2.03 \pm 0.15 \times 10^{88}$
Relic Neutrinos	$1.93 \pm 0.15 \times 10^{88}$
WIMP Dark Matter	$6 \times 10^{86 \pm 1}$
Relic Gravitons	$2.3 \times 10^{86^{+0.2}_{-3.1}}$
ISM and IGM	$2.7 \pm 2.1 \times 10^{80}$
Stars	$3.5 \pm 1.7 \times 10^{78}$
Total	$\mathbf{2.6 \pm 0.3 \times 10^{122}}$
Tentative Components:	
Massive Halo BHs ($10^{5} M_{\odot}$)	10^{104}
Stellar BHs (42 – 140 $M_{\odot}$)	$1.2 \times 10^{98^{+0.8}_{-1.6}}$

NOTE. — This budget is dominated by the cosmic event horizon entropy. While the CEH entropy should be considered as an additional component in scheme 2, it also corresponds to the holographic bound ('t Hooft 1993) on the possible entropy of the other components and may represent a significant overestimate. Massive halo black holes at $10^{5} M_{\odot}$ and stellar black holes in the range 42 – 140 $M_{\odot}$ are included tentatively since their existence is speculative.

Other big/small numbers in Physmatics/Chemistry

- Graviton-Graviton conversion into photons: $\sigma \sim 10^{-110} cm^2$.
- The informational Bekenstein bound will be about $2{,}6 \cdot 10^{42}$ bits and represents the maximal information needed to perfectly recreate an average human brain down to the quantum level. This means that the number $O = 2^I$ of states of the human brain must be less than $\approx 10^{7{,}8 \times 10^{41}} \approx 10^{8 \times 10^{41}} \sim 10^{10^{42}}$.
- Bohr radius: $10^{-10} m$.
- $k_w(25^\circ C) = 10^{-14}$.
- Hidróxido de talio(III): $K_s(25^\circ C, Th(OH)_3) = 1{,}68 \cdot 10^{-44}$, sulfuro de mercurio(II) rojo: $K_s(HgS) = 2 \cdot 10^{-54}$.
- For the reaction $3I_2(s) + 2Al(s) \rightarrow 2Al^{3+}(aq) + 6I^-(aq)$, the so called Nernst equation gives you $K_{redox,eq} = 10^{223}$. Equilibrium constant of the cell's redox reactions are generally big numbers.
- $H_2(g) \leftrightarrows 2H(g)$ has $K_c = 10^{-36}$ at $25^\circ C$ but it is only $6 \cdot 10^{-5}$ at $5000^\circ C$.

Contenido

Otras dimensiones y geometría

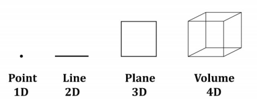
Geometría

Parte de las Matemáticas que estudia las propiedades puras de los objetos y figuras abstractos. Vacío, puntos, líneas, superficies, volúmenes, hipervolúmenes...Se corresponden al vacío, partícula, vector, bivector, trivector o sólido, y los hipersólidos.

Dimensión

Una defición simple de dimensión está relacionada con la sucesión del número de puntos mínimo para tener una figura:

$$-1, 0, 1, 2, \ldots, D-1 = N \leftrightarrow 0, 1, 2, 3, \ldots, D = N+1$$



Otras dimensiones y geometría(II)

- Una dimensión en matemáticas es generalmente un número entero positivo o nulo. Wait a moment! ¿Podría haber dimensiones negativas?
- Si la dimensión es no entera, tenemos fractales o incluso multifractales si la dimensión no entera no es constante.
- Si la dimensión es negativa, hay objetos exóticos con dimensiones negativas en Topología.
- Si la dimensión es compleja $D_C = 2D_R$, si es cuaterniónica $D_Q = 4D_R = 2D_C$, y si es octoniónica $D_O = 8D_R = 2D_Q = 4D_C$.

Objetos geométricos dimensiones para $D \in \mathbb{Z}^+$:

- Puntos, $\sum$(Punto): conjunto discreto(periódico, cuasiperiódico, aperiódico), conjunto contínuo. Redes (lattices): conjunto discreto de punto con cierta (cuasi)periodicidad/aperiodicidad u orden. En Física producen cristales, cuasicristales, policristales,...
- Líneas, $\sum$(líneas): contínuo (espacio-tiempo), discreto (polígonos, grafos o haces/rayos). Si incluimos orientabilidad tenemos digrafos o multidigrafos, suma de líneas/puntos dirigidos.

Objetos geométricos(I)

- Si de un punto a otro punto hay 1 línea tenemos un grafo. Si de un punto a otro hay 0 o múltiples líneas, tenemos un multigrafo.
- Si incluimos orientaciones: digrafos y multidigrafos.
- Si hay infinitas líneas orientadas, tenemos un apeiro(di)grafo.
- Grafos conocidos: puntos, polígonos, poliedros, politopos.
- Grafos no muy conocidos: apeirógonos, apeiroedros, apeirotopos. Reciente interés en apeirotopos conocidos como Amplituedros, Asociaedros y EFTedros en Física.
- Objetos con singular geometría: polígonos y poliedros regulares. Existe una clasificación de poliedros regulares interesante según las dimensiones.
- Nombre usuales: vacío(reposo), punto(cero, singularidad), línea(segmento), polígonos regulares(triángulos equiláteros, cuadrados, pentágonos, n-ágonos regulares), poliedros regulares platónicos(tetraedro, cubo o hexaedro, octaedro, dodecaedro, icosaedro), poliedros cruzados,. . .

Objetos geométricos(II)

- Con dimensión -1, hay $0, 1, \infty$? tipos de politopos negativodimensionales.
- Con dimensión 0 y 1, solamente hay 1 politopo regular (el punto).
- Con $d = 2$: los politopos regulares son los polígonos regulares.
- Con $d = 3$: hay esencialmente 5=1+4 poliedros regulares o platónicos (tetraedro o símplex, cubo o hexaedro, octaedro, dodecaedro e icosaedero).
- Con dimensión 4: hay 6 politopos 4d regulares,
  - 5-cell C_5(célula de 5 cuerpos, 4-simplex, pentácoro, pentatopo, pentaedroide), C_8 8-cell(célula de 8 cuerpos, octácoro, octaedroide, tetracubo, prisma cúbico,**teseracto**, hipercubo).
  - 16-cellC_{16}(hexadecácoro, hexadecaedroide, célula de 16 cuerpos, politopo cruzado, cocubo, 4-orthoplex), 24-cell C_{24}(célula de 24 cuerpos, icositetrácoro, octoplex, hiperdiamante, polioctaedro).
  - 120-cellC_{120}(célula de 120 cuerpos o caras, dodecahedral complex, dodecaplex, polidodecaedro, hecatonicosácoro, dodecacontácoro, hecatonicosaedroide), 600-cellC_{600}(célula de 600 cuerpos, hexacosicoro, hexacosiedroide).

- En dimensión arbitraria d o nd, con $d = D = N \geq 5$ solamente hay 3 politopos regulares: el n-cube, el n-orthoplex y el n-simplex.
- Un n-cube/n-cubo (cuadrado, cubo, teseracto, penteracto,...) contiene 2^n puntos o vértices, y tiene $2n$-caras. Para un n-cubo hay un número de m-cubos($m < n$) enla frontera del $C_n(\partial C_n)$.

Fórmulas del n-cubo

El hipervolumen V_n, la hipersuperficie total S_n y el número de m-cubos($m < n$) de un n-cubo regular de lado L se determina con las fórmulas:

$$V_n = L^n \quad S_n = (2n)L^{n-1} \quad E_{m,n} = 2^{n-m}\binom{n}{m} \tag{13}$$

y donde se define $\binom{n}{m} = \dfrac{n!}{(n-m)!m!}$. Un n-cubo posee 2^n vertices, $2^{n-1}n$ edges, $2^{n-3}(n-1)n$ faces, y 2n-cells.

Objetos geométricos(IV)

- En dimensión arbitraria d o nd, con $d = D = N \geq 5$ solamente hay 3 politopos regulares: el n-cube, el n-orthoplex y el n-simplex.
- Un n-orthoplex, hiperoctaedro, dual del n-cubo, es un politopo cruzado con $2n$ puntos o vértices y 2^n caras. Para un n-orthoplex:

Fórmulas del n-orthoplex o hiperoctaedro

El hipervolumen V_n, la hipersuperficie total S_n y el número de k-dim($k < n$) de un n-orthoplex regular de lado L se determina con las fórmulas:

$$V_n = \frac{\sqrt{2^n}}{n!}L^n \quad S_n(op) = \frac{\sqrt{2^{n+1}n}}{n!}L^{n-1} \quad E_{k,n} = 2^{k+1}\binom{n}{k+1} \tag{14}$$

Un politopo cruzado n-dimensional, o hiperoctaedro, tiene $2n$ puntos $2(n-1)n$ edges, $\frac{4}{3}(n-2)(n-1)n$ caras y 2^n cuerpos(cells).

Objetos geométricos(V)

- En dimensión arbitraria d o nd, con $d = D = N \geq 5$ solamente hay 3 politopos regulares: el n-cube, el n-orthoplex y el n-simplex.
- Un n-simplex, hipertetraedro, $n+1$ puntos o vértices y $(n+1)n/2$ caras. Para un n-simplex:

Fórmulas del n-simplex o hipertetraedro

El hipervolumen V_n, la hipersuperficie total S_n y el número de k-dim($k < n$) caras de un n-simplex regular de lado L se determina con las fórmulas:

$$V_n(\Delta) = \frac{\sqrt{2^{-n}(n+1)}}{n!}L^n = \frac{\sqrt{(n+1)}}{n!2^{n/2}}L^n \quad S_n(\Delta) = \frac{\sqrt{2^{1-n}n}}{(n-1)!}(n+1)L^{n-1}$$

$$\mathrm{E}_{k,n} = \binom{n+1}{k+1} \quad (15)$$

Un n-simplex tiene $n+1$ vértices, $n(n+1)/2$ edges, $(n+1)n(n-1)/6$ caras y $n+1$ cells o cuerpos.

Objetos geométricos(VI): la hiperesfera(I)

Hiperesfera

Hiperesfera(euclidiana) o n-esfera es el lugar geométrico S^n (n-1)-dimensional asociado a la ligadura $\sum_{i=1}^{n} x_i^2 = x_1^2 + \cdots + x_n^2 = R^2$.

Hipervolumen e hipersuperficie de la hiperesfera de radio R

El hipervolumen $V(S^n)$ y la hipersuperficie $\Sigma(S^n)$ se calcula con:

$$V_n = \frac{\Gamma(1/2)R^n}{\Gamma\left(\frac{n}{2}+1\right)} \quad \Sigma_n = \frac{dV_n}{dR} = \frac{n\Gamma(1/2)R^{n-1}}{\Gamma\left(\frac{n}{2}+1\right)} = \frac{2\Gamma^{(n+1)}(1/2)R^n}{\Gamma\left(\frac{n+1}{2}\right)} \tag{16}$$

y donde $\Gamma(1/2) = \sqrt{\pi} = (-1/2)!$. Interesante: $V(S^\infty) = 0$, y el volumen de la 23-esfera unidad es igual a la densidad del retículo de Leech $\Lambda_{24} = \pi^{12}/12!$ que subyace a la simetría del grupo monstruo M.
Recurrencia dimensional: $V_n = \dfrac{\Sigma_{n-1}}{n}$.

Objetos geométricos(VI): la hiperesfera(II)

$$V(S^0) = 2R \tag{17}$$

$$V(S^1) = \pi R^2 \approx 3{,}14159 R^2 \tag{18}$$

$$V(S^2) = \frac{4}{3}\pi R^3 \approx 4{,}11879 \tag{19}$$

$$V(S^3) = \frac{\pi^2}{2} R^4 \approx 4{,}9348 R^4 \tag{20}$$

$$V(S^4) = \frac{8\pi^2}{15} R^5 \approx 5{,}26379 R^5 \tag{21}$$

Objetos geométricos(VI): la hiperesfera(II)

$$V(S^5) = \frac{\pi^3}{6}R^6 \approx 5{,}16771R^6 \tag{17}$$

$$V(S^6) = \frac{16\pi^3}{105}R^7 \approx 4{,}72477R^7 \tag{18}$$

$$V(S^7) = \frac{\pi^4}{24}R^8 \approx 4{,}05871R^8 \tag{19}$$

$$V(S^8) = \frac{32\pi^4}{945}R^9 \approx 3{,}29851R^9 \tag{20}$$

$$V(S^9) = \frac{\pi^5}{120}R^{10} \approx 2{,}55016R^{10} \tag{21}$$

Objetos geométricos(VI): la hiperesfera(II)

$$V(S^{10}) = \frac{64\pi^5}{10395} R^{11} \approx 1{,}8841 R^{11} \quad (17)$$

$$V(S^{11}) = \frac{\pi^6}{720} R^{12} \approx 1{,}33526 R^{12} \quad (18)$$

$$V(S^{12}) = \frac{128\pi^6}{135135} R^{13} \approx 0{,}919629 R^{13} \quad (19)$$

$$V(S^{13}) = \frac{\pi^7}{5040} R^{14} \approx 0{,}599265 R^{14} \quad (20)$$

$$V(S^{14}) = \frac{256\pi^7}{2027025} R^{15} \approx 0{,}381443 R^{15} \quad (21)$$

Objetos geométricos(VI): la hiperesfera(II)

$$V(S^{15}) = \frac{\pi^8}{40320} R^{16} \approx 0{,}235331 R^{16} \qquad (17)$$

$$V(S^{16}) = \frac{512\pi^8}{34459425} R^{17} \approx 0{,}140981 R^{17} \qquad (18)$$

$$V(S^{23}) = \frac{\pi^{12}}{479001600} R^{24} \approx 0{,}00192957 R^{24} \qquad (19)$$

$$V(S^{24}) = \frac{8192\pi^{12}}{7905853580625} R^{25} \approx 0{,}000957722 R^{25} \qquad (20)$$

$$V(S^{25}) = \frac{\pi^{13}}{6227020800} R^{26} \approx 0{,}000466303 R^{26} \qquad (21)$$

$$V(S^{26}) = \frac{16384\pi^{13}}{213458046676875} R^{27} \approx 0{,}000222872 R^{27} \quad (17)$$

$$V_{91} \begin{cases} \dfrac{\pi^{46} R^{92}}{55026221598120889498503054288002548929616517529600000000000} \\ \approx 1{,}34377 \cdot 10^{-35} R^{92} \end{cases} \quad (18)$$

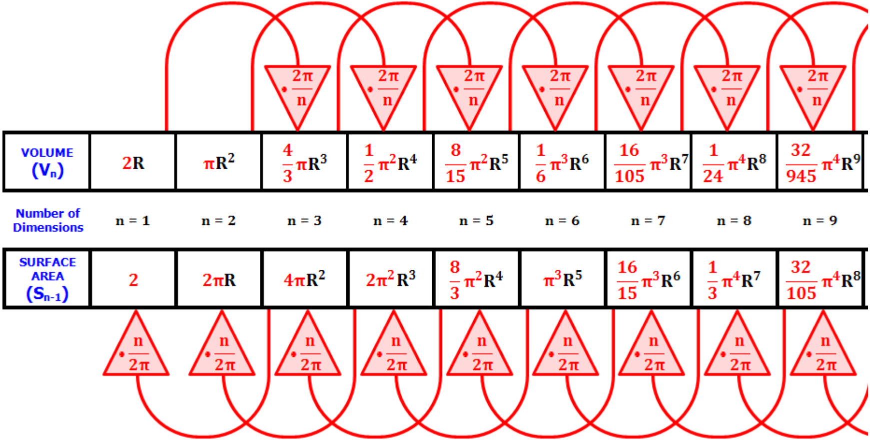
And finally, two more...The 4096-dimensional sphere

$$V(S^{4095}) \approx 8{,}70008138919055 \times 10^{-4877} R^{4096} \quad (19)$$

with a fantastic fraction that can not be written in the margin or space of this page easily. The final one, surprisingly, the infinite-dimensional sphere volume is zero:

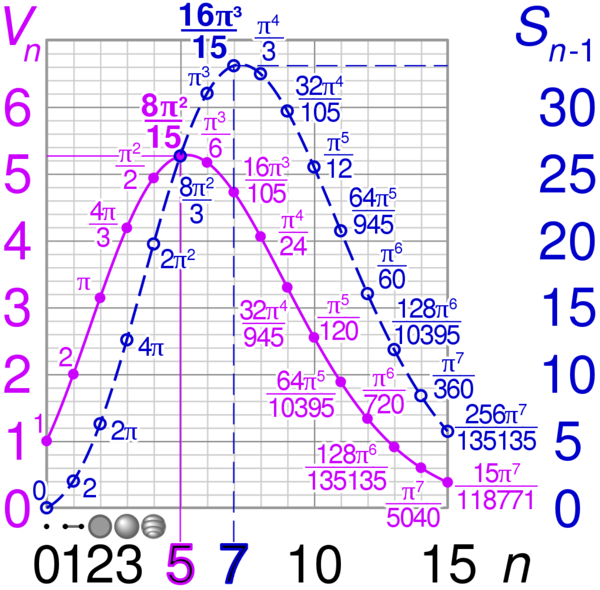
$$V(S^{\infty}) = 0 \tag{17}$$

The amazing vanishing sphere volume with increasing dimensions!!!!!!!!!!

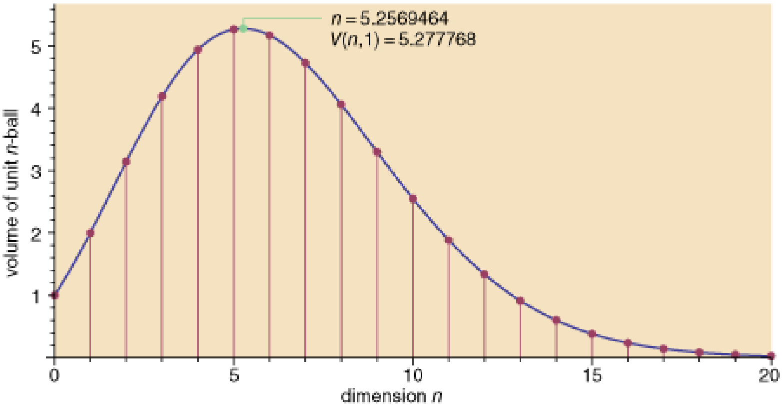


VOLUME (V_n)	$2R$	πR^2	$\frac{4}{3}\pi R^3$	$\frac{1}{2}\pi^2 R^4$	$\frac{8}{15}\pi^2 R^5$	$\frac{1}{6}\pi^3 R^6$	$\frac{16}{105}\pi^3 R^7$	$\frac{1}{24}\pi^4 R^8$	$\frac{32}{945}\pi^4 R^9$
Number of Dimensions	n = 1	n = 2	n = 3	n = 4	n = 5	n = 6	n = 7	n = 8	n = 9
SURFACE AREA (S_{n-1})	2	$2\pi R$	$4\pi R^2$	$2\pi^2 R^3$	$\frac{8}{3}\pi^2 R^4$	$\pi^3 R^5$	$\frac{16}{15}\pi^3 R^6$	$\frac{1}{3}\pi^4 R^7$	$\frac{32}{105}\pi^4 R^8$

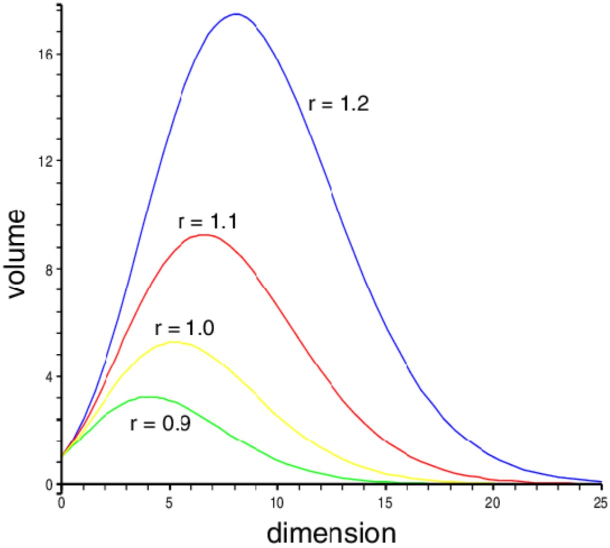
Objetos geométricos(VI): la hiperesfera(II)



Objetos geométricos(VI): la hiperesfera(II)



Objetos geométricos(VI): la hiperesfera(II)

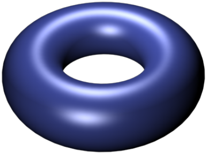


Objetos geométricos(VII): volúmenes y áreas en 2d/3d

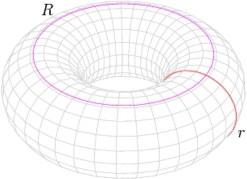
- $A_\Delta = bh/2$. Cuadrado y rectángulo: $A_L = L^2$, $A_R = ab$.
- Área del polígono regular de n-lados de longitud L:
  $A = pa/2 = \frac{1}{4} nL^2 \cot\left(\frac{\pi}{n}\right)$, $p = nL$.
- Tetraedro: $V_T = \frac{L^3}{6\sqrt{2}} = \frac{\sqrt{2}L^3}{12}$, $S_T = \sqrt{3}L^2$.
- Cubo: $V = L^3$, $S = 6L^2 = 3!L^2$. Octaedro: $V = \frac{\sqrt{2}}{3}L^3$, $S = 2\sqrt{3}L^2$.
- Dodecaedro: $V = \frac{1}{4}\left(15 + 7\sqrt{5}\right)L^3$, $S = 3\sqrt{5(5+2\sqrt{5})}L^2$.
- Icosaedro: $V = \frac{5}{6}\phi^2 L^3 = \frac{5}{6}\left(\frac{1+\sqrt{5}}{2}\right)^2 L^3$, $S = 5\sqrt{3}L^2$.
- Prism(cuboid): $V = abc$, $S = 2(ab + bc + ac)$.
- Cono y pirámide cuadrada: $V_C = \pi R^2 h/3$, $A_C = \pi R\sqrt{R^2 + h^2}$,
  $V_P = A_b h/3$, $A_P = B + PL/2$, P es el perímetro y $L = \sqrt{h^2 + r^2}$.

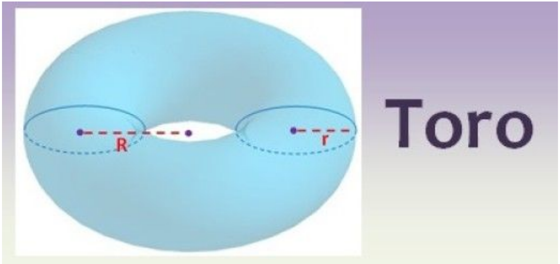
Objetos geométricos(VIII): el toro

- Dónut o toro: $V_{toro} = (\pi r^2)(2\pi R) = 2\pi^2 R r^2$, donde R es la distancia del centro del toro al centro del tubo, y r es el radio del tubo. El área del toro es igual a $A_{toro} = (2\pi r)(2\pi R) = 4\pi^2 R r$.



$$\left(R - \sqrt{x^2 + y^2}\right)^2 + z^2 = r^2$$





Objetos geométricos(IX): volúmenes y áreas en 4d

- 4-cell(hipertetraedro): $= \frac{\sqrt{5}L^4}{96}$, $S = \frac{5L^3}{6\sqrt{2}}$.
- 8-cell(teseracto,hipercubo): $V = L^4$, $S = 8L^3$.
- 16-cell(4-orthoplex,hiperoctaedro,cocubo,politopo cruzado/cross polytope): $V = \frac{L^4}{6} = \frac{L^4}{3!}$, $S = \frac{4\sqrt{2}}{3}L^3$.
- 24-cell(hiperdiamante): $V = 2L^4$, $S = 8\sqrt{2}L^3$.
- 120-cell: $V = \frac{15}{4}(105 + 47\sqrt{5})L^4$, $S = 30(15 + 7\sqrt{5})L^3$.
- 600-cell: $V = \frac{25}{4}\left(2 + \sqrt{5}\right) L^4$, $S = 50\sqrt{2}L^3$.
- Para $D = -1, 0, 1, 2, 3, 4, \geq 5$ hay $\infty?, \infty?, 1, \infty, 5, 6, 3$ politopos regulares.
- La $(n+2)$-esfera $S \times S^n$ es tal que $S^0 = \pm R$, $S^1 = S = U(1) = RP^1 = SO(2)$, $S^2 = CP^1 = SO(3)/SO(2)$, $S^3 = Sp(1)$ (glome, glomo) es igual a $Spin(3) \simeq SU(2) \simeq SO(4)/SO(3)$ a nivel de grupos abstractos.

Objetos geométricos(X): esferas y esferas exóticas

- 4-esfera $HP^1 \simeq SO(5)/SO(4)$.
- 5-esfera $U(1)$ sobre CP^2, $SO(6)/SO(5) \simeq SU(3)/SU(2)$.
- 6-esfera. Octoniones puros (estructura casi compleja): $SO(7)/SO(6) \simeq G_2/SU(3)$.
- 7-esfera. Cuasigrupo topológico de octoniones unitarios. $Sp(1)$ sobre S^4. $SO(8)/SO(7) \simeq SU(4)/SU(3) \simeq Sp(2)/Sp(1)$. Simetría especial $Spin(7)/G_2 \simeq Spin(6)/SU(3)$. Hay esferas exóticas en 7d.
- 8-esfera es OP^1. 23-esfera está relacionada(como mencionamos) con el retículo o red de Leech Λ_{24}

Esferas exóticas

Se llama esfera exótica a toda variedad o espacio diferenciable homeomorfo pero no difeomorfo a la n-esfera.

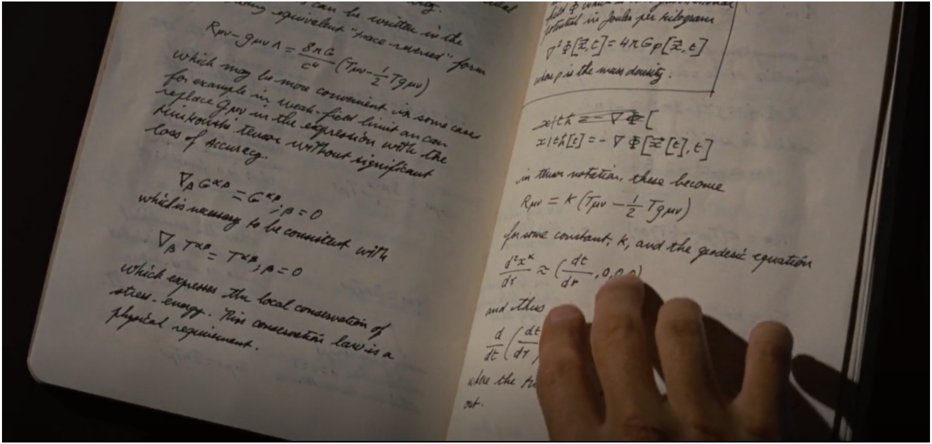
Objetos geométricos(XI): Otros objetos

- Amplituedro (Física): cierto politopo (apeirótopo) que contiene poliedros y el denominado grassmanniano positivo como subespacios y que codifica las amplitudes de scattering de procesos físicos simplificando el cálculo de los antiguos diagramas de Feynman.

$$\boxed{\mathcal{M}_{n,k,L}\left[Z_a\right] = \mathrm{Vol}\left[\mathcal{A}_{n,k,L}\left[Z_a\right]\right]}$$

- Asociaedro (Matemáticas, K_n): politopo convexo $(n-2)$-dimensional, en el que cada vértice corresponde a una forma de insertar paréntesis (y) en una palabra de n letras, y los lados a una aplicación de la regla de asociatividad. Sinónimo de asociaedro es politopo de Stasheff. Nima A.H. relaciona asociaedros y amplituedros con amplitudes y cinématica o dinámica de scaterring de partículas elementales o cuerdas.
- Pistas y relaciones entre Geometría abstracta(discreta y combinatoria) con Geometría física de altas energías y en el contínuo. Similar relación a la conjetura del brillo lunar (Moonshine conjecture): relación entre grupos finitos simples y formas modulares(funciones de variable compleja con ciertas propiedades).

Meet Tony Stark studying his father notebook!



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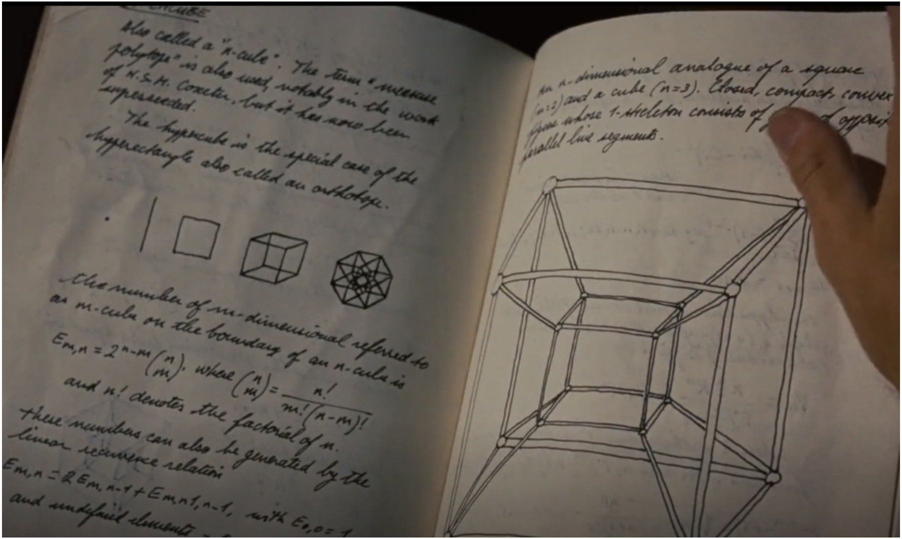

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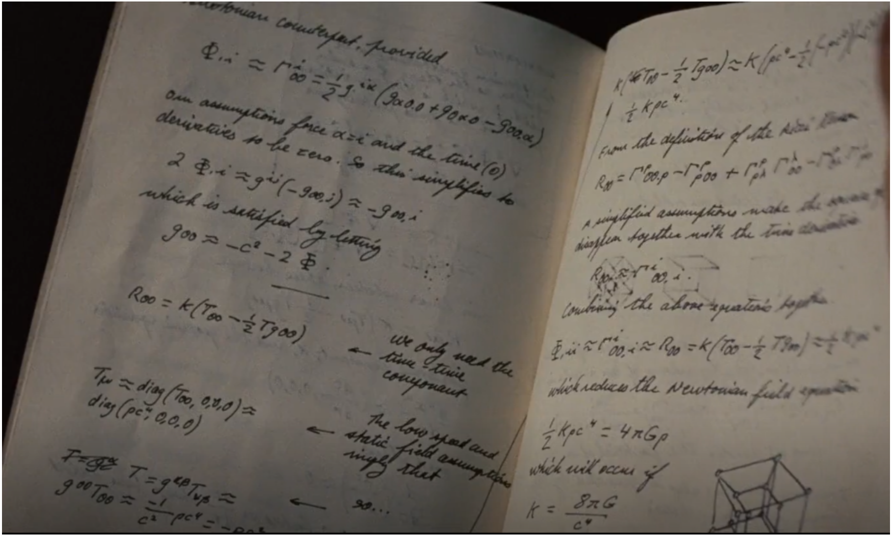

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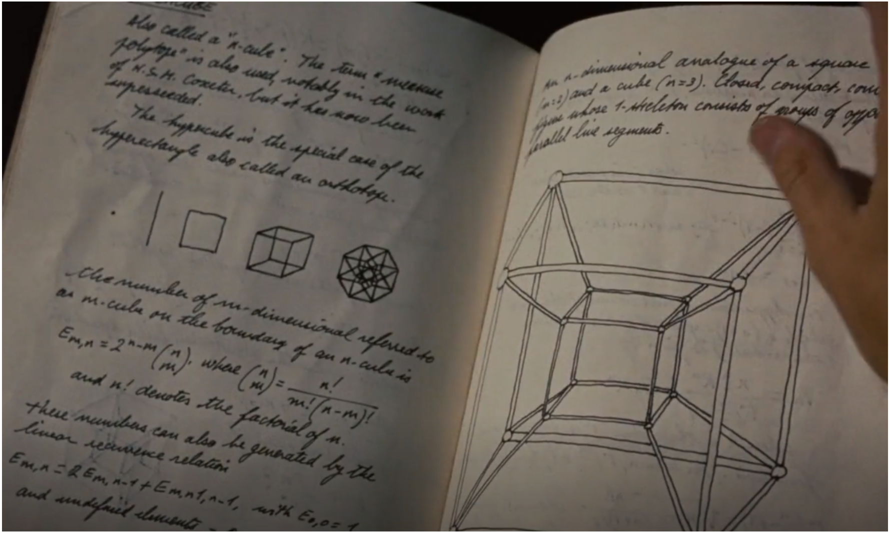

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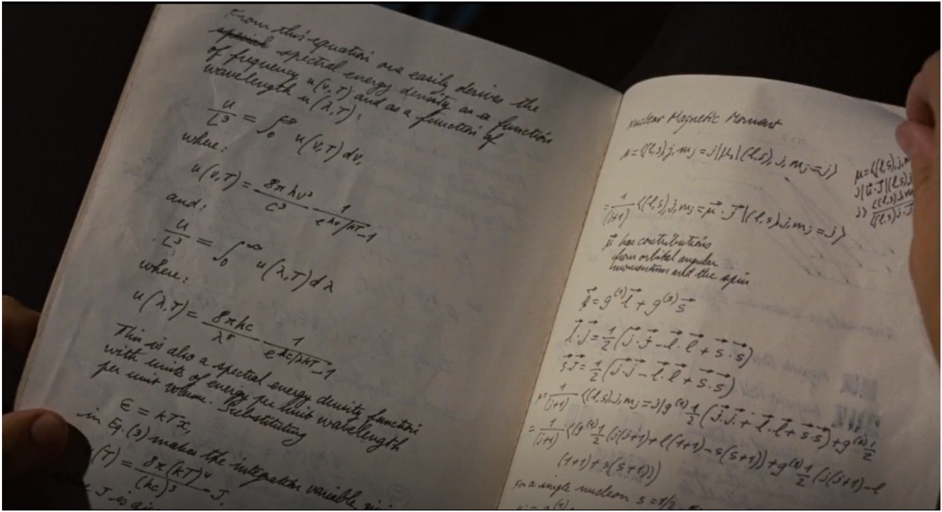

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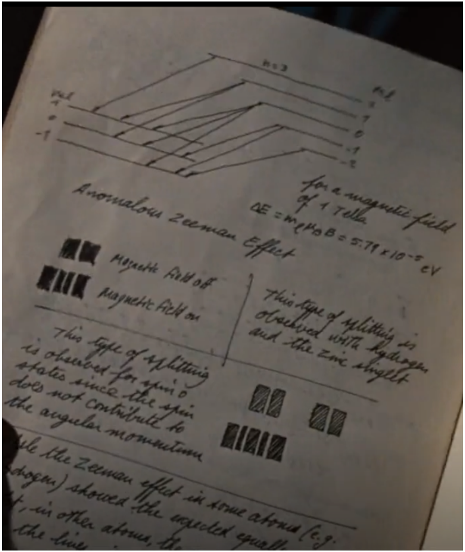

El efecto Zeeman anómalo es cuántica y geometría.

Contenido

Ecuaciones favoritas(I)/Favourite equations(I)

- **Relatividad especial.**
  $E = Mc^2$, $E = m\gamma c^2$, $E_0 = mc^2$, $\Delta E = \Delta Mc^2$.
  $\gamma = \left(1 - \frac{v^2}{c^2}\right)^{1/2}$, $\beta = v/c$, $E^2 = p^2c^2 + m^2c^4$, $p = Mv = m\gamma v$.
- **Física cuántica**.
  $E = hf = \hbar\omega$, $p = \hbar k = \frac{hc}{\lambda}$, $p = E/c$, $\lambda_{dB} = \frac{h}{p}$.
- **Función zeta de Riemann**.
  $\zeta(s) = \sum_{n=1}^{\infty} n^{-s} = \prod_{p=2}^{\infty}(1 - p^{-s})^{-1}$.
  $\zeta(s) = 0 \leftrightarrow s = -2n, n \in \mathbb{Z}, s_n = 1/2 \pm i\lambda_n$ (Hipótesis de Riemann).
- **Ecuaciones de campos**.
  $\Delta\Psi(x,t) = 0$. $\Box^2\phi(x,t) = 0$. $(\Box^2 + m^2c^2/\hbar^2)\phi = 0$.
  $(i\gamma \cdot D - mc/\hbar)\Psi = 0$.
- **Ecuaciones de Maxwell**.
  $F = dA$. $dF = 0$, $\star d \star F = \delta F = 4\pi j$.$[F = d_D A, \star d \star F = -\Omega_D j_D]$.
  $\partial_\mu F^{\mu\nu} = j^\nu$.$\varepsilon^{\mu\nu\sigma\tau}\partial_\nu F_{\sigma\tau} = 0$.

Ecuaciones favoritas(II)/Favourite equations(II)

- **Ecuaciones de Euler-Lagrange.**
  $\delta S=0$, $S=\int L dt$, $S=\int \mathcal{L} d^D x$, $E(L)=0$.
  $E(L)=\dfrac{\partial L}{\partial x}-\dfrac{d}{dt}\left(\dfrac{\partial L}{\partial \dot{x}}\right)=0$. $E(L)=\dfrac{\partial L}{\partial \varphi}-\dfrac{d}{dx^\mu}\left(\dfrac{\partial L}{\partial \partial_\mu \varphi}\right)=0$.
- **Ondas planas.**
  $\vec{E}=\vec{\varepsilon}e^{i(\omega t-kr)}$. $\vec{B}=\vec{\epsilon}e^{i(\omega t-kr)}$. $h_{\mu\nu}=E_{\mu\nu}\exp(i(\omega t-kr))$.
- **Ecuaciones de Yang-Mills.**
  $F^a_{\mu\nu}=\partial_\mu A^a_\mu-\partial_\nu A^a_\mu+gf^a_{bc}\left[A^b_\mu,A^c_\nu\right]$.
  $DF=0$ $(D^\mu F_{\mu\nu})^a=0.[D_A F=0, \star D_A\star F=-\Omega_D J_D]$.
  Bianchi: $D_\mu F_{\nu\kappa}+D_\kappa F_{\mu\nu}+D_\nu F_{\kappa\mu}=0$. $\partial^\mu F^a_{\mu\nu}+gf^{ab}_c A^\mu_b F^c_{\mu\nu}=0$.
- **Dualidad.**
  $\tilde{F}^{\mu\nu}=\dfrac{1}{2}\varepsilon^{\mu\nu\sigma\tau}F_{\sigma\tau}$. $D_\mu\tilde{F}^{\mu\nu}=0$.
- **Dimensiones de los campos.**
  $\left[A^a_\mu\right]=L^{\frac{2-D}{2}}=\sqrt{L^{2-D}}$. $g^2_{YM}=L^{D-4}$,$g_{YM}=\sqrt{L^{D-4}}$. $\alpha=\dfrac{g^2}{4\pi}$ adimensional.

- **Ecuación del grupo de renormalización.**

  Función beta: $\beta(g)=\mu^2\frac{\partial g}{\partial \mu^2}=\frac{\partial g}{\partial \log \mu^2}$

  $\beta(\alpha_s)(SU(N))=-\frac{11N}{12\pi}\alpha_s^2-\frac{17N^2}{24\pi^2}\alpha_s^3+\mathcal{O}(\alpha_s^4)$ implica que $\beta(\alpha_s)=\frac{\alpha_s^2}{\pi}\left(-\frac{11N}{6}+\frac{N_F}{3}\right)$. El Modelo Estándar tiene $SU(3)$, $N_c=3$, y $N_F=6$, con lo cual $\beta(g)<0$ y tenemos el fenómeno denominado libertad asintótica de quarks. En general, la negatividad de la función beta implica, bajo hipótesis de libertad asintótica, una relación

  $$-\frac{11N_c}{6}+\frac{N_F}{3}<0\rightarrow N_F<\frac{11N_c}{2}$$

  El número máximo de sabores para 3 colores es 16.

- Transformaciones de gauge.

  $$\delta A_\mu=\partial_\mu\varphi$$

Ecuaciones favoritas(IV)/Favourite equations(IV)

- **Efecto Schwinger gravitacional y eléctrico.**

$$A=\frac{mc^3}{\hbar},\quad E=\frac{m^2c^3}{q\hbar}$$

- **Radio de Schwarzschild y longitud, tiempo y energía de Planck.**
$R_s=2GM/c^2$, $L_p^2=G\hbar/c^3$, $t_p=L_p/c$, $E_p=\sqrt{\dfrac{\hbar c}{G_N}}$.
- **Absement espacio-temporal.**
$\mathcal{A}^\mu=\int\varphi(x)dx^\mu$. $\mathcal{A}^\mu=\int X^\mu d\tau$.
$\mathbb{A}^\mu=\int\mathcal{A}^\mu d^dx=\int X^\mu d\tau d^dx=\int X^\mu dvol$.
- **Ecuaciones de campo de la relatividad general.**
$G_{\mu\nu}+\Lambda g_{\mu\nu}=8\pi G_N T_{\mu\nu}$ $(c=1)$.
- **Perturbaciones gravitacionales débiles.**

$$g_{\mu\nu}=\eta_{\mu\nu}+h_{\mu\nu}$$

Ecuaciones favoritas(V)/Favourite equations(V)

- **Ondas gravitacionales débiles (lineales).**

$$\bar{h}_{\mu\nu} = A_{\mu\nu} e^{ixp}$$

- **Mecánica cuántica.**

$E\Psi = H\Psi$. $H = T + U$. $p = i\hbar\partial$. $[x, p] = i\hbar$.

$\left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)\Psi = 0$. $\Psi = \prod_i \Psi(x_i)$. $[x, x] = [p, p] = 0$.

$\Delta A \Delta B \geq \frac{1}{2}\langle [A, B]\rangle$, $\Delta X \Delta P \geq \hbar/2$. $\Delta E \Delta t \geq \hbar/2. \hbar = h/2\pi$.

- **Física básica de agujeros negros.**

$S_{BH} = k_B \frac{A}{4L_p^2}$, $S_{BH} = \frac{k_B c^3 A}{G\hbar}$, $S = k_B \log \Omega$, $dS/dt \geq 0$,

$dS/dt \geq k_B$. $S \geq k_B$. $S \geq \hbar$. $v \leq c$. $\Delta A = n\kappa L_p^2$

$T_{BH} = \frac{\hbar c^3}{8\pi G_N M k_B}$. $t_{ev} = \frac{5120\pi G^2 M^3}{\hbar c^4}$. $\tau_{XD} \sim \frac{1}{M_\star}\left(\frac{M_{BH}}{M_\star}\right)^{\frac{n+3}{n+1}}$.

$P_{BH} = L_{BH} = \frac{\hbar c^6}{15360\pi G^2 M^2}$.

Ecuaciones favoritas(VI)/Favourite equations(VI)

- **Agujeros negros microscópicos (primordiales,...)**: $T \leq 2{,}73K \rightarrow M_{BH} \leq 5 \cdot 10^{22} kg$ (now, ligeramente menor que la masa lunar).
- **Átomo de Bohr**. $L = mvr = n\hbar$, $r_n = a_0 n^2$. $E_n = Ry\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$. $E_n = -\frac{Ry}{n^2}$.
- **Desintegraciones radioactivas. Ley de desintegración**. $N = N_0 e^{-\lambda t}$. $t(1/2) = \frac{\ln 2}{\lambda}$. $N = N_0 2^{-t/t_{1/2}}$. $\tau = \frac{1}{\lambda} = \frac{t_{1/2}}{\ln 2}$. $\Gamma\tau \geq \hbar/2$.
- **Polilogaritmos**.

$$Li_s(z) = \sum_{k=1}^{\infty} \frac{z^k}{k^s}, \quad Li_s(1) = \zeta(s)$$

$$Li_{s+1}(z) = \int_0^z \frac{Li_s(t)}{t} dt$$

Ecuaciones favoritas(VII)/Favourite equations(VII)

- Ecuaciones de Friedman($P=\omega\rho$):

$$H^2=\frac{8\pi G_N}{3}\rho-\kappa\frac{c^2}{R}+\frac{\Lambda}{3}\qquad\frac{\ddot{R}}{R}=-\frac{4\pi}{3}G_N\rho+\frac{\Lambda c^2}{3}$$

- Dirac quantization for the magnetic monopole:

$$\text{Gaussian units: }\frac{2Q_eQ_m}{\hbar c}\in\mathbb{Z},\quad\text{SI units,Weber conv.:}\frac{Q_eQ_m}{2\pi\hbar}\in\mathbb{Z},$$

$$\text{SI,A-m: }\frac{2Q_eQ_m}{4\pi\varepsilon_0\hbar c^2}\in\mathbb{Z},\text{ or SI,A-m:}\frac{2K_CQ_eQ_m}{\hbar c^2}\in\mathbb{Z}$$

- T, S dualities:
$L\leftrightarrow\frac{\alpha'}{L},(n,\omega)\leftrightarrow(\omega,n).\alpha\leftrightarrow 1/\alpha, Q_e\leftrightarrow Q_m,\phi\leftrightarrow 1/\phi, e\leftrightarrow 4\pi\hbar c/e.$
- Complex spectra: $M^2=\alpha Q_e^2+Q_m^2/\alpha$, $\tau=\frac{\theta}{2\pi}+i\frac{2\pi}{e^2}$, $\tau'=-1/\tau$,
$M=\langle\phi\rangle\sqrt{q^2+m^2}$. Born reciprocity: $p\rightarrow q, q\rightarrow -p$.
$E\rightarrow B, B\rightarrow -E$.

Contenido

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Баярлалаа
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kia ora
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misaotra
matondo
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Hypothesis:

- Magnetic and electric field of a point monopole charge with $Q_m = g$ and dual charge $e_g = eg/c = egv/c^2$.

$$F_e + F_m = 2F_{m,e} = F_c \leftrightarrow \frac{2K_c e_g}{R^2} = \frac{mv^2}{R} \rightarrow \frac{c^{-2}eg}{4\pi\varepsilon_0} = \frac{mvR}{2} = \frac{n\hbar}{2}$$

Then, $eg = \dfrac{n\hbar c^2}{2K_C}$(Q.E.D.). Equivalently: $\dfrac{g}{e} = \dfrac{nc}{2\alpha_e} \leftrightarrow \alpha_e = \dfrac{nce}{2g}$

- Dirac-Zwanziger-Schwinger dyonic quantization $Z = (e, g)$:

$$e_1 g_2 - e_2 g_1 = 2\pi n\hbar c$$

From this, it follows that $Q = ne$, or $Q = \left(n + \dfrac{1}{2}\right) e$ and

$$M = \sqrt{\frac{K_C}{G_N}}e = \frac{\hbar c^2}{g}\sqrt{\frac{1}{K_C G_N}}.$$

The existence of magnetic monopoles implies the quantization of Q_e.

Bohr-logy in XD

2d Bohr energy levels and radius

In any 2d Universe, the Bohr-Rydberg energy and radius are:

$$E = Ke^2\left(\frac{1}{2} + \ln(n)\right), \quad r_n = na_0(2d) = \frac{n\hbar}{\sqrt{mKe}}, n \in \mathbb{Z} \qquad (17)$$

For gravitational case, take $Ke^2 \rightarrow GMm$.

Dd Bohr energy levels and radius

In any Dd Universe, the Bohr-Rydberg energy and radius are:

$$E = \frac{D-4}{2(D-2)}\left(\frac{m}{\hbar^2}\right)^{\frac{D-2}{4-D}} n^{-\frac{2D-4}{4-D}} e^{\frac{4}{4-D}}, \quad r_n(D) = \left(\frac{m}{\hbar^2}\right)^{\frac{1}{D-4}} e^{\frac{2(2-D)}{(4-D)(D-2)}} n^{\frac{2}{4-D}} \qquad (18)$$

For gravitational case, take $Ke^2 \rightarrow GMm$.

Gravity in XD

Newton in higher dimensions

In any Dd ($D = d + 1$) Universe (spacetime), the gravitational force, the gravitational field, the potential energy and the potential read

$$F_N = G_D \frac{Mm}{r^{D-2}} = G_{d+1} \frac{Mm}{r^{d-1}} \quad g = G_D \frac{M}{r^{D-2}} = G_{d+1} \frac{M}{r^{d-1}} \tag{19}$$

$$U_g = G_D \frac{Mm}{r^{D-3}} = G_{d+1} \frac{Mm}{r^{d-2}}$$

$$V_g = G_D \frac{2\Gamma((D-1)/2)M}{\pi^{(D-3)/2} r^{D-3}} = G_{d+1} \frac{2\Gamma(d/2)M}{\pi^{(d-2)/2}(d-2)r^{d-2}} \tag{20}$$

Dilution of gravity: $G_N(4d) = G_D/V_D$. $g^2_{YM}(4d) = g^2_{YM,d} R^{-d}$,

$M_P = \sqrt{hc/G} \sim 10^{-5} g, M_W = \frac{h}{c}\sqrt{\Lambda/3} \sim 10^{-65} g. Gh\Lambda/c^3 \sim 10^{-121}$.

$M_U = \frac{c^2}{G}\sqrt{3/\Lambda} \sim 10^{56} g$, $M'_W = \sqrt[3]{\frac{h^2\sqrt{\Lambda/3}}{G}} \sim 10^{-25} g$. $M_U/M_W \sim 10^{121}$

Uniform sphere total energy in XD

Gravitational/electric energy for uniform density sphere

$$U_g = -G_{d+1}\frac{d(d-2)M^2}{d+2}\frac{1}{R^{d-2}} = -G_D\frac{(D-1)(D-3)M^2}{D+1}\frac{1}{R^{D-3}} \quad (21)$$

with $D = d+1$ and M the mass. If $M = \rho V$, then

$$U_g = -G_{d+1}\frac{d(d-2)\pi^d\rho^2}{(d+2)\Gamma^2\left(\frac{d}{2}+1\right)}R^{d+2} = -G_D\frac{(D-1)(D-3)\pi^d\rho^2}{(D+1)\Gamma^2\left(\frac{D+1}{2}+1\right)}R^{D+1} \quad (22)$$

Trickery for the electric case: substitute $G_n \to K_C$, $M \to Q$, with $Q = \rho V$.

Entropic gravity in XD

Hypothesis for $D = d + 1$ hyperdimensional Newton gravity:

- $A(\Sigma) = \dfrac{2\pi^{d/2}R^{d-1}}{\Gamma(d/2)}$.
- $N = A(\Sigma)/L_p^{d-1}$, $E = mc^2 = Nk_B T/2$, $\Delta S = 2\pi k_B \dfrac{mc\Delta x}{\hbar}$.

Then:

$$F = -T\frac{\Delta S}{\Delta x} = -G_d\frac{Mm}{R^{d-1}}$$

where

Hyperdimensional gravitational Newton constant

$$G_d = \frac{2\pi^{1-d/2}\Gamma\left(\frac{d}{2}\right)c^3 L_p^{d-1}}{\hbar} = 2\pi^{1-d/2}\Gamma\left(\frac{d}{2}\right)\frac{c^3 L_p^{d-1}}{\hbar}$$

$$\phi_g = -\Omega_d G_d M; \quad \phi_e = \Omega_d K_d Q = Q/\varepsilon_0(d) \;\; \Omega_d = 2\pi^{d/2}/\Gamma(d/2)$$

Take the functional equation:
$\zeta(s) = 2^s \pi^{s-1} \sin\left(\frac{\pi s}{2}\right) \Gamma(1-s) \zeta(1-s)$ for $1-s = d/2$. Then, since

$$G_d = 2\pi^{1-d/2}\Gamma\left(\frac{d}{2}\right)\frac{c^3 L_p^{d-1}}{\hbar}$$

you can derive that

Gravitational constant and zeta function

$$G_d = \frac{\pi 2^{d/2}\zeta\left(1-\frac{d}{2}\right)}{\zeta\left(\frac{d}{2}\right)\cos\left(\frac{\pi d}{4}\right)}\left(\frac{c^3 L_p^{d-1}}{\hbar}\right) \tag{23}$$

Classical atom instability

Hypothesis: non quantum atoms are unstable. To prove this:

$$P = \frac{dE}{dt} = \frac{2e^2a^2}{3c^3}\text{(Larmor formula)}$$

$$\frac{Ke^2}{R^2} = \frac{mv^2}{R} \rightarrow v^2 = \frac{Ke^2}{mR},\ E = \frac{mv^2}{2} - \frac{Ke^2}{R} = -\frac{Ke^2}{R}$$

$$dt = -\frac{1}{\frac{dE}{dt}}\frac{dE}{dR}dR = -\frac{3}{16}\frac{m^2c^3R^2dR}{(E_0R_0)^2} \rightarrow \int_0^{t_c} dt = -\frac{3m^2c^3}{(E_0R_0)^2}\int_{R_0}^0 R^2dR$$

We finally get:

Decay time of classical em-atoms

$$t_c = \frac{m^2c^3R_0}{16E_0^2} = \frac{m^2c^3R_0^3}{4K_Ce^4} = \frac{4\pi^2\varepsilon_0^2m^2c^3R_0^3}{e^4} \simeq 1{,}6\cdot 10^{-11}s \sim 20ps$$

Gravitational music equations

Binary system with $M = M_1 + M_2$, $f_{GW} = 2f_{orb}$, $M_c = (M_1 M_2)^{3/5}/M^{1/5}$ yields (GR):

$$L_{GW} = \frac{2^5}{5}\left(\frac{G^{7/3}}{c^5}\right)[M_c \pi f_{GW}]^{10/3}$$

$$\dot{f}_{GW} = \left(\frac{96}{5}\right)\left(\frac{G^{5/3}}{c^5}\right)\left(\pi^{8/3}\right)(f_{GW})^{11/3}$$

$$t_c = \frac{2}{2^8}\left(\frac{GM_c}{c^3}\right)^{-5/3}[\pi f_{GW}]^{-8/3}$$

Neutrino oscillations: the equations

Supposing transitions between different neutrino species, via $|\nu_\alpha\rangle$ to $|\nu_\beta\rangle$

Neutrino oscillations

$$\mathcal{A}=P(\alpha\rightarrow\beta)=|\langle\nu_\beta|\nu_\alpha\rangle|^2=\left|\sum_j U^*_{\alpha j}U_{\beta j}e^{-i\frac{m_j^2c^3L}{2\hbar E}}\right|^2 \quad (24)$$

$$\mathcal{A}=\delta_{\alpha\beta}-4\sum_{i>j}\text{Re}\left(U^*_{\alpha i}U_{\beta i}U_{\beta j}U^*_{\beta j}\right)\sin^2\left(\frac{\Delta m^2_{ij}c^3L}{4\hbar E}\right)+$$

$$+2\sum_{i>j}\text{Im}\left(U^*_{\alpha i}U_{\beta i}U_{\beta j}U^*_{\beta j}\right)\sin\left(\frac{\Delta m^2_{ij}c^3L}{2\hbar E}\right) (25) \quad \text{with}$$

Neutrino transition matrix

$$|\nu_\beta\rangle=U^\alpha{}_\beta|\nu_\alpha\rangle$$

Multitemporal physics(I)

Usual 1T newtonian physics: $F = ma = m\dfrac{dv}{dt} = m\dfrac{d^2r}{dt^2} = -\nabla U(r)$, assuming conservative forces only. Let $W = F_i dx^i$ the work form, in a ND manifold $V \subset \mathbb{R}^N$, with submanifold nd $M \subset \mathbb{R}^n \subset \mathbb{R}^N$. $y^I = y^I(x)$, $\omega = f_I dy^I$ implies $dy^I = \dfrac{\partial y^I}{\partial x^i}dx^i$, and also

$$W = F_i(x)dx^i \rightarrow F_I = f_I(y(x))\frac{\partial y^I}{\partial x^i}$$

Single time manifold approach

$$f_I = m\delta_{IJ}\frac{d\dot{y}^J}{dt} = m\delta_{IJ}\frac{d^2y^I}{dt}$$

$$F_i = m\delta_{IJ}\frac{d\dot{y}^I}{dt}\frac{\partial y^J}{\partial x^i} = m\delta_{IJ}\frac{d^2y^I}{dt^2}\frac{\partial y^J}{\partial x^i}$$

Multitemporal physics(II)

Going multitemporal with timelike coordinates $(t) = t^\alpha$, $\alpha = 1, \cdots, m$

Multitime tensorial Newton 2nd law

$$f_I = m_{IJ}\delta^{\alpha\beta}\frac{\partial^2 y^J}{\partial t^\alpha \partial t^\beta}$$

$$f_i = m_{IJ}\delta^{\alpha\beta}\frac{\partial^2 y^I}{\partial t^\alpha \partial t^\beta}\frac{\partial y^J}{\partial x^i}$$

with anti-trace $F_i = F^\alpha_{i\alpha}$ given by the tensor 1-form

$$F^\sigma_{i\alpha} = m_{IJ}\delta^{\sigma\beta}\frac{\partial^2 y^I}{\partial t^\alpha \partial t^\beta}\frac{\partial y^J}{\partial x^i}$$

(Multitime) Kinetic energy

$$T = E_k = \frac{1}{2}m\delta_{IJ}\dot{y}^I\dot{y}^J \quad T = \frac{1}{2}\delta_{IJ}\delta^{\alpha\beta}\frac{\partial y^I}{\partial t^\alpha}\frac{\partial y^J}{\partial t^\beta}$$

Multitemporal physics(III)

Single time Euler-Lagrange 1st order EOM

$$\delta S = 0 \rightarrow E(L) = \frac{\partial L}{\partial x^i} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}^i}\right) = 0$$

(Multitime) Euler-Lagrange EOM

$$\delta S = 0 \rightarrow E(L) = \frac{\partial L}{\partial x^i} - D_\alpha\left(\frac{\partial L}{\partial D_\alpha x^i}\right) = 0$$

(Multitime) Euler-Lagrange EOM: nth order

$$E(L) = \sum_{j=0}^{n}(-1)^j\left(\frac{\partial L}{\partial \partial_t^j x^i}\right) = 0 \quad E(L) = \sum_{J=0}^{n}(-1)^J\left(\frac{\partial^J L}{\partial D_\alpha^J x^i}\right) = 0$$

Multitemporal physics(IV)

Single time Hamilton EOM

Define 1T hamiltonian as $H = \dot{x}^i \frac{\partial L}{\partial \dot{x}^i} - L$, and $p_i = \partial L / \partial \dot{x}^i$, then

$$\dot{x}^i = \frac{dx^i}{dt} = \frac{\partial H}{\partial p_i} \qquad \dot{p}_i = \frac{dp_i}{dt} = -\frac{\partial H}{\partial x^i}$$

Multi-time Hamilton EOM

Define nT hamiltonian as $H = D_\alpha x^i \frac{\partial L}{\partial D_\alpha x^i} - L$, and $p_i^\alpha = \partial L / \partial D_\alpha x^i$, then

$$\frac{\partial x^i}{\partial t^\alpha} = \frac{\partial H}{\partial p_i^\alpha} \qquad \frac{\partial p_i^\beta}{\partial t^\alpha} = -\delta^\beta{}_\alpha \frac{\partial H}{\partial x^i}$$

Astrometry

$$\theta = \left(\frac{M_p}{M_\star}\right)\left(\frac{a}{r}\right) \approx \frac{10^{-3}}{r(pc)}\left(\frac{P(yr)}{M_\star(\odot)}\right)^{2/3} M_p(J)$$

Here

$$V_r(m/s) \approx \frac{30}{(P(yr))^{1/3}} \frac{M_p(J)}{M_\star(\odot)^{2/3}} \sin(i)$$

Detecting exoplanets(II)

Microlensing

$$R_E^2 = \frac{4GMD}{c^2}, \quad D = \frac{D_{ds} D_d}{D_s}, \quad t_0 = \frac{R_E}{v_e}$$

$$t_0 = \frac{2 D_L \theta_E}{v_L} = \frac{2\theta_L}{v_L}\sqrt{\frac{4GM(1 - D_s/D_s)}{c^2 D_d}}$$

The impact parameter u reads

$$A = \frac{u^2 + 2}{u(u^2 + 4)^{1/2}}$$

Detecting exoplanets(III)

Direct detection

$$B \geq \frac{\lambda D}{r} \approx \left(\frac{\lambda}{10\mu mu}\right)\left(\frac{D}{10pc}\right)\left(\frac{r}{1AU}\right)^{-1} m$$

Detecting exoplanets(IV)

Radial velocity

$$K_\star = \left(\frac{2\pi G_N}{P}\right)^{1/3} \frac{M_p\,(M_\star + M_p)^{1/3}\sin(i)}{M_\star} \frac{1}{\sqrt{1-e^2}}$$

Also, it is usually written with $M_\star + M_P \simeq M_\star$ as follows

$$M_p \sin(i) = \left(\frac{P}{2\pi G}\right)^{1/3} K_\star M_\star^{2/3}\sqrt{1-e^2}$$

Mathematical challenge

Solve, exactly, the following equations:

- $xe^x = z$
- $xa^x = Y$
- $x^n e^{x^n} = Z$
- $x^n a^{x^n} = Y$
- $\ln(A + BX) + CX = \ln(D)$
- $x^{x^{x^{\cdots}}} = y$
- $A^x = Bx + c$

Doctor Strange in the Multiverse of Madness!