Physics concepts

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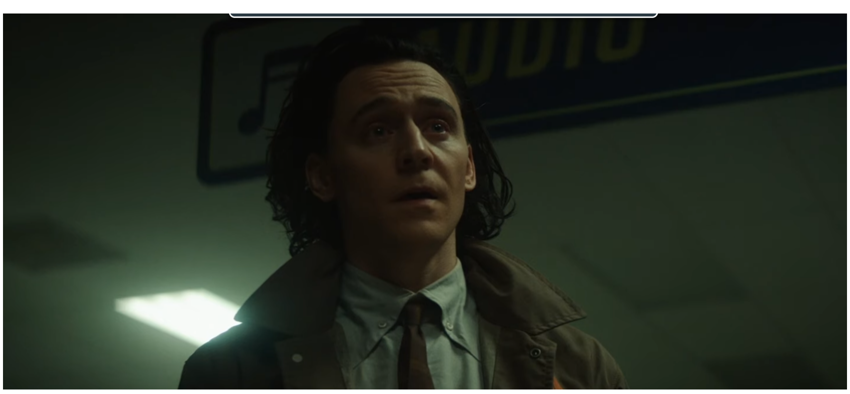
Abstract

A summary of preuniversitary Physics and general Physics.



Contents

1 The Physical World

1.1 Fundamental forces of Nature

Gravitacional force

- Force of mutual attraction between any two objects by virtue of their masses.
- Key role: the largest scales (and likely, smallest scales).
- The phenomena in the Universe, such as formation of and evolution of stars, galaxies and galactic clusters, are due to gravitational forces.
- $F_N = G_N \frac{Mm}{r^2}$, newtonian gravitational force.
- Quantum gravity should be mediated by gravitons.

Electromagnetic force

- Force between electrical charges, static or moving.
- Both attractive and repulsive.
- About 10^{36} stronger than gravity when compared the electrical force and gravitational force of 2 protons, for any fixed distance.
- Coulomb law: $F_C = K_C \frac{Qq}{r^2}$.
- Magnetic force: $F_m = q\vec{v} \times \vec{B}$.
- Magnetic force for currents: $\vec{F} = K_m \frac{\oint \oint II' d\vec{l} \times (d\vec{l} \times \vec{r})}{r^3}$.
- Due to photons (massless).

Strong nuclear force

- Very short range, strongest force.
- Attractive in nature.
- Range $10^{-18} m$ or less.
- Strongest between all known fundamental forces.
- Due to gluon (massless) interchanges between quarks. Effective theory described but mesonic (massive) interchanges.

Weak nuclear force

- Very short range, $10^{-16}m$ or less, nuclear force.
- Responsible of radioactive decay and flavor changing.
- Not as weak as gravity but much weaker than strong nuclear force and electromagnetic forces (about $10^3 - 10^5$ times weaker than those).
- Due to exchanges of weak massive vector bosons W, Z.
- Indirectly points out to the existence of a scalar field responsible of their non-null gauge masses.
- Higgs field is really another field beyond the other 4, responsible of tuning the masses of fundamental particles that are elementary. Reason of mass: interactions with Higgs particles/field. Mechanism similar to superconductivity: spontaneous symmetry breaking.

1.2 Science principles and Technology

Science

- Physics: study of basic laws of Nature and manifestation in different natural phenomena.
- Physics is an experimental Science, but it uses Mathematics as core language.
- Principles: unification and reductionism. Unification tries to explain diverse physical phenomena in terms of same concepts and laws. Reductionism tries to derive the properties of bigger and more complex systems from the properties of its constituent or simpler parts.
- Branches: Classical Physics (macroscopic phenomena as Mechancis, Electrodynamics, Optics, Thermodynamics, Fluids,...), Quantum Physics (deals with microscopic phenomena at the smallest scales like atoms, molecules, nuclei or subatomic particles).
- Patterns: Any subject is said to be Science when it is studied in sequence patterns AND the hypothesis to be checked by experiments, logic or computational simuluations. Experimenting, observing, exploring, predicting, checking the data, elaboring theories are essential to the scientific method.

Science method and Technology

- Systematic observations, controlled experiments, qualitative and quantitative reasoning, mathematical modelling, prediction and verification or falsation of theories.
- Technology gives rise to new physics. Similarly, physics gives rise to new technologies.

2 Units and measurement

2.1 Significant figures

Numbers and figures

- Measurements are provided by numbers. Real numbers are generally used.
- The digits of a number that are used to express to the required degree of accuracy.
- We use scientific notation $x.y_1y_2\cdots 10^{\pm n}$.
- All non-zero digits are significative.
- All zero between two non-zero digits.
- If the number is less than 1, the zero(s) on the right of decimal point but to hte left of the non-digit zero are not significant.
- Terminal or trailing zero(s) in a number without a decimal point are not significant. E.g.: 020342.010 (7s.f.).

2.2 Units

Units and dimensions

- Comparison with a certain internationally accepted reference standard is called unit.
- Reference standard use to measure physical quantities.
- Expresion showing how and which of the base quantities represent the dimensions of a physical quantity is called dimension formula. E.g.: force=$[MLT^{-2}]$.

Measurement and experiments

- Measurement has precision and accuracy.
- Precision is the limit of resolution of measured quantity. Precision is related to the standard deviation of measurements and the limits of the experimental devices measuring magnitudes.
- Accuracy is the closeness of the measured value to the true value. Accuracy is computed as relative and absolute errors.

Errors

- Absolute error: $\Delta_a X = |x_i - \overline{x}|$. Average: $\overline{x} = \frac{\sum_i x_i}{n}$.
- Relative error: $\Delta_r X = \frac{\Delta_a X}{\overline{x}} \cdot 100$.
- Error combination: $\Delta Z = \Delta A \pm \Delta B$ for Z=$A \pm B$.
- Error combination(II): $\frac{\Delta Z}{Z} = \left(\frac{\Delta A}{A}\right) + \left(\frac{\Delta B}{B}\right)$ for $Z = A/B$ or $Z = AB$.
- Error combination(III): $\frac{\Delta Z}{Z} = p\left(\frac{\Delta A}{A}\right) + q\left(\frac{\Delta B}{B}\right) + r\left(\frac{\Delta C}{C}\right)$ for $Z = A^p B^q / C^r$.
- Error combination: average error. $\varepsilon(\overline{x}) = \frac{\sigma(x_i)}{\sqrt{n}}$
- Error combination(V): A much better combination is taking quadratures on sigmas or individial errors of the average.

Systems of units

- Fundamental units (S.I.): Mass, Length, Time, Temperature, amount of substance, electric current, luminous intensity. Units: kilogram, meter, second, kelvin, mole, ampere, candela.
- The units that are not basic are derived from basic units. E.g.: speed, acceleration, force, pressure,...
- Quantities like plane angle, solid angle, have radian and stereradian as units but are dimensionless.
- Properties of matter and energy require some universal constants: Universal acceptance, non-perishable, well defined, does not change with time or space.
- Common system of units: CGS (cm, g, s), FPS (foot, pound, second), MKS (metre, kilogram, second), S.I.

3 Motion(I)

Straight motion

- Average velocity: $\vec{v}_M = \frac{\Delta \vec{r}}{\Delta t}$.
- Instantaneous velocity $\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \lim_{\Delta t \to 0} \vec{v}_M = \frac{d\vec{r}}{dt}$.
- Average acceleration: $\vec{a}_M = \frac{\Delta \vec{v}}{\Delta t}$.
- Instantaneous acceleration $\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \lim_{\Delta t \to 0} \vec{a}_M = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(\frac{d\vec{r}}{dt}\right) = \frac{d^2\vec{r}}{dt^2}$.

Relative velocity and uniform motion

- Relative velocity vector $\vec{v}_{AB} = \vec{v}_B - \vec{v}_A$.
- Uniform motion equations: $\vec{a} = \overrightarrow{const.}$, $\vec{v} = \vec{v}_0 + \vec{a}\Delta t$, $\vec{r} = \vec{r}_0 + \vec{v}_0 \Delta t + \frac{1}{2}\vec{a}\Delta t^2$, $\vec{v}^2 = \vec{v}_0^2 + 2\vec{a} \cdot \Delta \vec{r}$.

4 Motion(II): plane motion

Vectors and components

- Plane vectors: $\vec{v} = v_x\vec{i} + v_y\vec{j}$. Polar coordinates: $v_x = v\cos\theta$, $v_y = v\sin\theta$, $v^2 = v_x^2 + v_y^2$.
- Space vectors (3d): $\vec{v} = v_x\vec{i} + v_y\vec{j} + v_z\vec{k}$. Director angles: $v_x = v\cos\gamma$, $v_y = v\cos\beta$, $v_z = v\cos\gamma$, with $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$.
- Intrinsic coordinates in the plane: $\vec{v} = v\vec{\tau}$, $\vec{a} = a_n\vec{n} + a_\tau\vec{\tau}$. $a_n = \dfrac{v^2}{R}$, $a_\tau = \dfrac{dv}{dt}$, $a^2 = a_n^2 + a_\tau^2$.
- Modulus of a vector 2d: $v = +\sqrt{v_x^2 + v_y^2}$. Modulus of a vector 3d: $v = +\sqrt{v_x^2 + v_y^2 + v_z^2}$.
- Free fall: $y = y_0 - \dfrac{gt^2}{2}$.
- Parabolic motion(ballistics): $x = (v_0\cos\theta)t$, $y = (v_0\sin\theta)t - \dfrac{1}{2}gt^2$.
- Path equation for parabolic motion: $y = x\tan\theta - \dfrac{g}{2v_0^2\cos^2\theta}x^2$. Flight time: $T = \dfrac{2v_0\sin\theta}{g}$.
- Parabolic motion: a) Maximum height: $y_m = \dfrac{v_0^2\sin^2\theta}{2g}$, b) Maximum range: $x_m = \dfrac{v_0^2\sin(2\theta)}{g}$.

5 Motion(III): Newton laws of motion

Newton laws

- First law: $\dfrac{\vec{F}_1}{a_1} = \dfrac{\vec{F}_2}{a_2} = \cdots = \dfrac{\vec{F}_n}{a_n} = m$. There are inertial frames where the same laws of motion holds.
- Second law: $\sum \vec{F}_i = \dfrac{d\vec{p}}{dt} = m\vec{a}$.
- Third law: $\vec{F}_{ij} = -\vec{F}_{ji}$.
- Centripetal force: $F_c = \dfrac{mv^2}{R}$.
- Conservation of momentum: $\vec{P} = \sum_i \vec{p}_i$ whenever no net force is applied. $\vec{p} = m\vec{v}$.
- Static friction: $F_r = \mu_s N$. Kinetic friction: $F_d = \mu_d N$. $F_R < F_k < F_s$.
- Car on banked force: $v_m = \left(Rg\dfrac{\mu_s + \tan\theta}{1 - \mu_s\tan\theta}\right)^{1/2}$. No friction: $v_m = Rg\tan\theta$.

6 Work, energy and power

Work and energy

- $dW = \vec{F} \cdot d\vec{r}$. $W = \int_C \vec{F} \cdot d\vec{r}$.
- For $\vec{F} = m\vec{a}$, dW=ΔE_c. $E_c = \frac{1}{2}mv^2 = \frac{p^2}{2m}$, kinetic energy.
- Mechanical energy: $E_m = E_c + E_p$.
- Potential energy: $\vec{F} = -\nabla E_p$.
- Mass-energy equivalence: $E = mc^2$, $\Delta E = \Delta mc^2$.
- Elastic potential energy: $E_p(el) = \frac{kx^2}{2}$. $F = -kx$, Hooke law.
- Gravitational potential energy: $E_p(g) = -\frac{GMm}{r}$, $E_p(g) = mgh$ at low height.
- Electric potential energy: $U_e = \frac{K_C Qq}{r}$.
- Generalized mechanical energy theorem: $\Delta E_m = W(F_r)$.

Power

- Power: $P = \frac{dW}{dt} = \vec{F} \cdot \vec{v}$.
- Units of Power: watts (1W=1$J \cdot s^{-1}$). Horse power (imperial) 746W. Horse Power (metrical) 735.5W. 1kWh=3.6MJ.

7 Systems of particles and rotational motion

Rotational uniform motion

- $\theta = \theta_0 + \omega_0 \Delta t + \frac{\alpha \Delta t^2}{2}$.
- $\omega = \omega_0 + \alpha \Delta t$.
- $a = \alpha R$, $a_c = \omega^2 R$. $a_t = R\sqrt{\alpha^2 + \omega^4}$.
- Torque: $\vec{\tau} = \vec{M} = \vec{r} \times \vec{F}$. $\sum_i \vec{\tau}_i = \frac{d\vec{L}}{dt} = I\alpha$.
- Angular momentum: $\vec{L} = \vec{r} \times \vec{p} = \overrightarrow{\overrightarrow{I}} \vec{\omega}$.
- Moment of inertia: $I = mr^2$. General: $I = \sum_i m_i r_i^2$. Radius of gyration: $k = \sqrt{I/m}$.
- Moment of inertia examples: $I(rod, cm) = ml^2/12$, $I(rod, edge) = ml^2/3$, $I(ring, cm) = mR^2$, $I(ring, dia) = mR^2/2$, $I(disk, cm) = mR^2/2$, $I(disk, dia) = mR^2/4$, $I(sphere) = 2MR^2/5$.
- Parallel theorem for MOI: $I_e = I(cm) + md^2$ (Steiner).
- Perpendicular theorem for MOI: $I_z = I_x + I_y$.

Systems of particles and rigid bodies

- Center of mass concept(discrete system): $\vec{r}_{cm} = \dfrac{\sum_i m_i \vec{r}_i}{\sum_i m_i}$.
- Center of mass concepto(continuous case): $\vec{r}_{cm} = \dfrac{\int \rho \vec{r} dV}{\int \rho dV}$.
- Rigid body: system of particles whose particle distance is left invariant during the motion.
- Velocity and acceleration for center of mass: $\vec{v}_{cm} = \dfrac{\sum_i m_i \vec{v}_i}{\sum_i m_i}$, $\vec{a}_{cm} = \dfrac{\sum_i m_i \vec{a}_i}{\sum_i m_i}$

8 Gravitation

Universal Force of Gravity

- $\vec{F}_N = -G_N \dfrac{Mm}{r^2}\vec{u}_r$. $\vec{g} = -\dfrac{G_N M}{r^2}\vec{u}_r$. $G_N = 6.674 \cdot 10^{-11} Nm^2/kg^2$.
- Universal gravity is central and conservative. $U_g = -\dfrac{G_N Mm}{r}$.
- Orbital energy: $E_m = -\dfrac{GMm}{2r}$. Desorbital/desatellization energy: $E_d = -E_m$.
- Satellization energy: $E_s = GMm_s\left(\dfrac{1}{r_0} - \dfrac{1}{2r(orb)}\right)$.
- Surface gravity: $g_s = \dfrac{GM}{r^2}$. $r = R_p + h$.
- Gravity variation with height: $g = g_0\left(1 - \dfrac{2h}{R_p}\right)$.
- Gravity variation with depth: $g = g_0\left(1 - \dfrac{d}{R_P}\right)$.
- Gravity variation with latitude and rotation: $g = g_0 - R\omega^2 \cos^2 \lambda$.

Satellites

- Geostationary satellites (T=T_p): for Earth $T = 24h$, $v = 3.1 km/s$, $r = 42400 km$, $h = 36000 km$.
- Polar satellites (T=84 minutes). $v = 7.92 km/s$, $h = 880 km$.
- Gravitational potential $V_g = E_p/m = -G\dfrac{M}{r}$.

Kepler laws

- Kepler first law: $r(\varphi) = \dfrac{\mathcal{P}}{1 + \varepsilon \cos(\varphi + \varphi_0)}$.
- Kepler second law: $V_A = \dfrac{|\vec{L}|}{2m} = \dfrac{vr}{2} = constant$.
- Kepler third law: $T^2 = kr^3$, $k = \dfrac{4\pi^2}{GM}$. $M = M_1 + M_2 = M_* + M_p$.

9 Mechanical properties of solids

9.1 Hooke's law, energy, stress and strain

Elasticity and modulus

- Hooke's law $F = -kx$ valid in the elastic limit, stress is directly proportional to strain, i.e, stress $\propto$ strain.
- Strain is the ratio of change in configuration to original configuration, i.e., $h = \frac{\Delta x}{x}$.
- Stress is the restoring force per unit area, i.e., $S = F/A$. Stress is a pressure.
- Types of stress: 1) Longitudinal $S_l = F_n/A$, 2) Volumetric $S_V = F_V/A$, and 3) Tangential stress or shearing $S_t = F_t/A$.
- Typos of strain: 1) Longitudinal $h_L = \frac{\Delta L}{L_0}$, 2) Volumetric $h_V = \frac{\Delta V}{V_0}$, and 3) Angular or shearing strain (angular displacement of the plane perpendicular to the fixed surface): $h_\theta = \frac{\Delta \theta}{\theta_0}$.
- Poisson's ratio sigma: $\sigma = \frac{\Delta d/d}{\Delta l/l}$ is the quotient between the lateral and longitudinal strain.

Bulk Modulus

- Young's modulus: $Y = \frac{F/A}{L/\Delta L} = \frac{F\Delta L}{AL} = \frac{Mg\Delta L}{\pi R^2 L}$.
- Bulk modulus or volume modulus of elasticity: $B = \frac{\Delta P}{\Delta V/V}$. The inverse $\frac{1}{B}$ measures compressibility, and B is roughly the hydraulic stress divided by the volume strain.
- Modulus of rigidity or shear modulus of elasticity is the quotient between the tangential stress and the shearing strain. Mathematically speaking: $\eta = \frac{A}{\theta} = \frac{F}{A\theta} = \frac{\sigma_s}{\varepsilon_s}$.

9.2 Elasticity

Elastic solids

- Elasticity is the property of solid materials making them regain their original shapes and size after a deformation force.
- Relations between solid modulus

$$Y = 3B(1 - 2\sigma) \tag{1}$$

$$Y = 2\eta(1 + \sigma) \tag{2}$$

$$\sigma = \frac{3B - 2\eta}{2\eta + 6B} \tag{3}$$

$$\frac{9}{Y} = \frac{1}{B} + \frac{3}{\eta} \quad \text{or} \quad Y = \frac{9B\eta}{\eta + 3B} \tag{4}$$

10 Mechanical properties of fluids

Bernouilli's principle

- For any incompressible, non-viscous, streamline, irrotational flow of fluid

$$P + \frac{1}{2}\rho v^2 + \rho g h = constant$$

- Bernouilli's principle applies to the Torricelli's law, giving the velocity of efflux of liquid through an orifice: $v = \sqrt{2gh}$.
- Bernouilli's principle applies to lift of an aircraft wing, the sprayer or atomizer and the blowing off the roofs during windstorm.
- The venturimeter is an application of Bernouilli's principle, being a device used to measure the rate of flow of liquid. The volume of liquid flowing per second is

$$Q = \frac{V}{s} = a_1 a_2 \sqrt{\frac{2h\rho_m g}{\rho(a_1^2 - a_2^2)}}$$

- Streamline: in liquid flow when the velocity is less than the critical velocity, each particle of the liquid passing through a point travels along the same path and same velocity as the preceding particles.
- Turbulence: when velocity of liquid flow is greater than critical velocity and partiles follow chaotic zig-zag paths.

Viscosity

- Opposing force between different layers of fluid in relative motion is a dragging force $F_d = -\eta A \frac{dv}{dx}$.
- η is the coefficient of viscosity.
- Surface tension is defined as $S = F/L$.
- Surface energy is defined as $S_E = \frac{dW}{dA}$.
- Capillary rise of fall, $h = \frac{2S\cos\theta}{r\rho g}$.
- Excess pressure inside a drop liquid is $P_e = \frac{2S}{R}$.
- Excess pressure inside a bubble (soap): $P_e = \frac{4S}{R}$.
- Fluid pressure: $P = \frac{dF}{dA} = \lim_{\Delta t \to 0} \frac{\Delta F}{\Delta A}$. Pressure exerted by a liquid column (or gaseous): $P = \rho g h$.
- Continuity equation: $m = a_1 v_1 \rho_1 = a_2 v_2 \rho_2$ for any incompressible liquid, $\rho_1 = \rho_2$, then $a_1 v_1 = a_2 v_2$.

Density and gauges

- Density: $\rho = M/V$. Specific density is the ratio of substance density with that of water at 4ºC ($1000 kg/m^3$).

Gauge pressure and other pressures

- Difference between the absolute pressure at a point and the atmospheric pressure: $\rho g\Delta h = P - P_a = \Delta P$.
- Total or actual pressure at a point: $P_x = P_a + \rho g h$.
- Pressure (atm) excerted by atmosphere. At sea level, 1 atm=pressure exerted by 0.76 m or 760 mmHg, 1013 mb or 101.3 kPa.
- Pascal's law: the pressure exerted at any point on an enclosed liquid is transmitted equally in all directions to all points. Hydraulic brakes and hydraulic lifts are based on Pascal's law: $P_1 = P_2$, $F_1/S_1 = F_2/S_2$.

11 Thermal properties of matter

Modes of heat transfer

- Conduction: heat transfer through molecular collisions without any actual motion of matter.
- Convection: heat transfer by actual motion of matter within the medium. Land breeze, sea breeze, trade winds based on natural convection are some examples.
- Radiation: heat transfer required no material medium, made by electromagnetic (gravitational, gluonic,...) radiation even in vacuum.

Thermal laws

- Stefan-Boltzmann law: for a blackbody with grey factors, energy per unit area and time is given by $E = \sigma e T^4$, with $\sigma = 5.67 \cdot 10^{-8} J s^{-1} m^{-2} K^4$, e is the emissivity.
- Perfect black body $e = 1$, thus $E = \sigma(T^4 - T_0^4)$.
- Wien's displacement law: $\lambda_m \propto 1/T$. $\lambda_m = b/T$. $b = 2.9 \cdot 10^{-3} mK$.
- Kirchoff's law: at any temperature, $e_\lambda / a_\lambda = E_\lambda = constant$.
- Newton's law of cooling: $\dfrac{dQ}{dt} = k(T_2 - T_1)$. For small temperature differences between a body and its surroundings, the loss of heat is given by the previous equation.
- Thermal conductivity: $\kappa = \dfrac{Qx}{A(T_2 - T_2)t}$.
- Heat is a form of energy, or more precisely, a form of transfer energy between systems, due to differences of temperature.
- Latent heat: heat required to change the state of unit mass substance with no temperature changes. $L = Q/m$. For water, $L_f = 3.33 \cdot 10^5 J/kg$ and $L_v = 22.6 \cdot 10^5 J/kg$.

Calorimetry

- Principle of calorimetry: $\Delta Q(lost) = -\Delta Q(gained)$.
- Specific heat capacity: $C_e = \dfrac{\Delta Q}{m\Delta T}$.
- Heat capacity: $\dfrac{\Delta Q}{\Delta T} = C$.
- Molar specific heat capacity: $s = \dfrac{\Delta Q}{n\Delta T}$.

Temperature and thermal expansions

- Thermometer: device to measure the degree of hotness or coldness of a body or system.
- Temperature scales:

$$\frac{T_C - 0}{100 - 0} = \frac{T_F - 32}{212 - 32} = \frac{T_K - 273.15}{373.15 - 273 - 15} = \frac{T_R - 0}{80 - 0}$$

- Thermal expansion is the increase (or decrease) in dimensions (physical) due to changes of temperature.
- Linear expansion:$\alpha = \frac{\Delta L}{L_0 \Delta T}$.
- Superficial expansion: $\beta = \frac{\Delta A}{A_0 \Delta T}$.
- Volume expansion: $\gamma = \frac{\Delta V}{V_0 \Delta T}$.
- Relationships between coefficient of expansions:

$$\gamma = 2\beta = 3\alpha$$

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- Young's modulus due to thermal stress: $\frac{\Delta F}{A} = Y\left(\frac{\Delta L}{L}\right)$.

12 Thermodynamics

Laws of thermodynamics

- 0th Law. If two systems (A,B) are in thermal equilibrium with a third system (C), then A and B are in equilibrium with each other, i.e. $T_A = T_B = T_C$.
- 1st Law. Energy is conserved and manifested as done work and heat: $\Delta U = \Delta Q + \Delta W = \Delta Q - P\Delta V$.
- 2nd Law. It is impossible for an engine working between a cyclic process to extract heat from a reservoir and convert it completely into work (Kelvin-Planck). No process is possible whose result is the absorption of heat from a reservoir and the complete transformation of heat into work. Clausius statement: no process is possible whose sole result is the transfer of heat from a colder object to a hotter object. Equivalently, entropy always increases or remains the same. $\delta S = \frac{\Delta Q}{T}$.
- 3rd Law. Measure of molecular disorder is the magnitude known as entropy, and it only goes to zero at absolute zero, but this is not reachable in any physical process in a finite number of steps. That is, zero absolute is not reachable.

State variables

- Thermodynamic state variables are of 2 types: 1) extensive (indicate the size of the system, like internal energy U, volume V, $mass$,...), and 2) intensive (do not indicate the size of the system, like pressure, temperature, ...).

Processes

- Any process made to proceed in the reverse direction by changing its conditions is called **reversible process**, and it is symmetrical with inversion of time. Any process which can not be retraced in the reverse direction exactly is called **irreversible process**.
- An ideal engine works on a reversible cycle of 4 operations: 1) isothermal expansion, 2) adiabatic expansion, 3) isothermal compression, 4) adiabatic compression. Efficiency of Carnot's engine:

$$\eta_C = \frac{W}{Q} = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1},\ \ T_2 < T_1$$

Thermodynamics and processes

- Thermodynamics is the branch of Physics dealing with concepts of heat and temperature, the conversion and transferring of heat and energy by thermodynamical processes.
- Isothermal process: satisfies $T = k$, $PV = const. = nRT$. Also: $S = \int_{V_i}^{V_f} nRT/VdV = nRT\ln(V_f/V_i)$.
- Adiabatic process: a thermally insulated system neither gains nor loses heat, and satisfies $PV^\gamma = const.$, $TV^{\gamma-1} = const.$, $P^{1-\gamma}T^\gamma = const.$, where $\gamma = C_P/C_V$ is the adiabatic coefficient. Work done adiabatically: $W = \dfrac{P_2V_2 - P_1V_1}{1-\gamma}$.
- Isochoric process: $V = k$. $P/T = k.$, $P_1/T_1 = P_2/T_2$.
- Isobaric process: $P = k.$, $V/T = k.$, $V_1/T_1 = V_2/T_2$.
- Cyclic process: any process in which a system returns to its initial state, and thus $\Delta U = 0$.
- Quasi-static process: infinity slow process such that the system reamins in thermal and mechanical equilibrium with the surroundings or environtment.
- Mayer's equation: $C_P - C_V = R$.

Refrigerator

- Refrigerator is a heat engine working in the reverse direction. Coefficient of performance:

$$\beta = \frac{Q_2}{W} = \frac{Q_2}{Q_2 - Q_1} = \frac{T_2}{T_2 - T_1},\ T_2 > T_1$$

- Refrigerator are time inverted Carnot's engine at most.

13 Kinetic theory

13.1 Ideal gases

Ideal gas

- An ideal gas satisfies the state equation $PV = nRT$ or $PV = Nk_BT$.
- Avogadro's law: $N_1/V_1 = N_2/V_2$.
- Boyle's law: $P_1V_1 = P_2V_2$.
- Charles' law: $V_1/T_1 = V_2/T_2$.
- Gay-Lussac's law: $P_1/T_1 = P_2/T_2$.
- Diver law: $P_1/n_1 = P_2/n_2$.
- No name law: $n_1T_1 = n_2T_2$.
- General laws: $PV/T = k$, $P/nT = k$, $V/nT = k$, $PV/n = k$.
- $R = N_Ak_B$ is an universal constant.
- Dalton's law of partial pressures: $P = P_1 + P_2 + \cdots P_n$, $P_i = \chi_i P_T$.

13.2 Kinetic theory and statistics

Statistical gases

- Pressure exerted by gas: $P = \dfrac{1}{3}\rho v_{rms}^2$.
- Average energy and pressure: $E = 3PV/2 = 3k_BTN/2$. $k_B = R/N_A$. $\overline{v} = v_{av} = \dfrac{v_1 + v_2 + \cdots + v_N}{N}$. $\overline{v} = \sqrt{\dfrac{8RT}{\pi \cdot MM}} = \sqrt{\dfrac{8}{3\pi}}v_{rms} = 0.92v_{rms}$.
- Most probable speed or velocity: $v_{mp} = \sqrt{\dfrac{2RT}{MM}} = \sqrt{\dfrac{2}{3}}v_{rms} = 0.816v_{rms}$.
- Root mean square speed or velocity:

$$v_{rms} = \sqrt{\frac{3PV}{M}} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3RT}{MM}}$$

Kinetic theory assumptions

- All the molecules of a gas are identical.
- The molecules of different gases are different.
- The molecules of gases are in a state of random motion.
- The collisions of gas molecules are perfectly ellastic.
- The interactions between molecules is due to electric or electromagnetic interactions only.
- The energy of every degree of freedom per molecule is $E/N = k_BT/2$ (equipartition theorem).
- Mean free path of molecules: $\lambda = \dfrac{1}{\sqrt{2}md^2} = \dfrac{k_BT}{\sqrt{2}\pi d^2 P}$.

Specific heat capacity for ideal gases

- For ideal gases: $C_P - C_V = R$.
- For monoatomic gases: $C_P/C_V = \gamma = 5/3$.
- For diatomic gases: $C_P/C_V = \gamma = 7/5$.
- For polyatomic gases: $C_P/C_V = \gamma = \frac{4+f}{3+f}$, where f is the degree of freedom of the gas.

14 Oscillations

14.1 SHO: Simple Harmonic Oscillators

Motion of simple oscillators

- SHO is the simplest form of oscillatory motion.
- Number of ocillations per second is frequency: $f = 1/T$. T is the period or smallest time interval after the motion is repeated.
- SHO has equation: $x(t) = A\cos(\omega t + \phi_0)$. Phase is $\omega t + \phi_0$.
- SHO velocity is $v(t) = dx/dt = -\omega A \sin(\omega t + \phi_0)$. $v_m = A\omega$.
- SHO acceleration is $a(t) = dv/dt = -A\omega^2 \cos(\omega t + \phi_0) = -\omega^2 x(t)$.
- SHO kinetic energy: $E_c = \frac{1}{2}m\omega^2 A^2 \sin^2(\omega t + \phi_0)$.
- SHO potential energy(elastic): $E_p = kx^2/2 = \frac{1}{2}m\omega^2 A^2 \cos^2(\omega t + \phi_0)$, $k = m\omega^2$ for SHO, since $F = -kx = ma$.
- Mechanical energy for SHO is constant: $E_m = kA^2/2 = \frac{1}{2}m\omega^2$.

Oscillations due to a spring

- $T = 2\pi\sqrt{\frac{m}{k}}$, where $k = F/\Delta x$.
- Simple pendulum: $T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{I}{mgL}}$, with $I = mL^2$.

14.2 Other types of oscillators

Damped oscillations

- Oscillations of a body whose amplitude goes on decreasing value with time. Damping force: $F_d = -bv$.
- Equation for damped simple oscillations:

$$x(t) = x_m e^{-bt/2m} \sin(\omega' t + \varphi_0)$$

  with

$$\omega' = \sqrt{\frac{m}{k} - \frac{b^2}{4m^2}}$$

- Amplitude is modulated and decreasing with time: $A(t) = x_m e^{-bt/2m}$.

Forced oscillations and resonances

- Driving force: $F(t) = F_0 \cos(\omega_d t)$.
- Displacement: $x(t) = A\cos(\omega t + \varphi_0)$.
- Amplitude for forced solutions is modulated as follows:

$$A = \frac{F_0}{\sqrt{m^2(\omega^2 - \omega_d^2)^2 + \omega_d^2 b^2}}$$

  and critical phase $\varphi_c = -v_0/\omega_d x_0$.

- When frequency ω_d is close to natural frequency of oscillator, amplitude blows out and increases strongly (resonance condition $\omega_d \simeq \omega$.

15 Waves

15.1 General properties and harmonic waves

Harmonic and plane waves

- Wave function or waveform for harmonic waves:

$$\Psi(x,t) = A\sin(\omega t \pm kx + \varphi_0)$$

- Alternative wavefunction formula:

$$\Psi(x,t) = A\sin\left[\left(\frac{2\pi x}{\lambda} \pm \frac{2\pi t}{t} + \varphi_0\right)\right]$$

- Propagation velocity $v_p = \lambda f = \dfrac{\omega}{k}$.
- Wave number and period-frequency equations: $\dfrac{2\pi}{\lambda} = k$, $\overline{k} = \dfrac{1}{\lambda}$, $\dfrac{1}{T} = f$, $\omega = 2\pi f$.
- Doppler effect:

$$f' = f_0\left(\frac{v \pm v_s \pm v_m}{v \pm v_{obs} \pm v_m}\right)$$

  where v is the speed of sound, v_{obs} the observer velocity, v_s the source speed or velocity and v_m the medium speed. With light (or alike), $v = c$ and $v_m = 0$ since the ether can not be detected and is non-physical.
- Difference in frequencies of two superposing waves generate a beat: $f_b = f_1 - f_2$.

15.2 Stationary waves

Stationary waves and properties

- In strings, fundamental frequency reads $f_0 = \dfrac{1}{2L_s}\sqrt{\dfrac{T_s}{m}}$.
- In organ pipes, open at both ends: fundamental frequency or harmonic reads $f_0 = v/2L$.
- In organ pipes, closed at one end, $f_0 = v/4L$.
- In open organ pipes, higher harmonics, both odd and even. In closed organ pines, higher harmonics, odd modes only.

15.3 Superposition and types of waves

Superposition principle

- Superposition principle for waves: $\Phi = \sum_i \Psi_i$.
- Matter waves associated with particles like electrons, protons, neutrons, molecules or rigid bodies (de Broglie hypothesis):

$$\lambda(dB) = \frac{h}{p} = \frac{h}{m\gamma v} \simeq \frac{h}{mv}$$

  where the last equality is valid in non-relativistic approximations ($v << c$).

Types of waves

- Mechanical waves are essentially featured by elasticity, inertia and minimum friction. They require a material medium.
- Electromagnetic, electroweak, strong force and gravitational waves do not require a medium for their propagation (even in vacuum).
- Individual particles of the medium oscillate perpendicularly to the direction of propagation if the wave is transversal.
- Individual particles of the medium oscillate longitudinally to the direction of propagation if wave is longitudinal.
- Longitudinal waves has speed: $v=\sqrt{\frac{B}{\rho}}$, with B the bulk modulus, or $v=\sqrt{\frac{Y}{\rho}}=\sqrt{\frac{\gamma P}{\rho}}$, where the last equality is for air with $\gamma = 7/5$.
- Transverse wave speed: $v=\frac{T}{m}$, where T is the tension and $m = M/L$ is the linear mass-density.

15.4 Other waves

Non-linear and dispersive waves

- There are non-linear waves that do not satisfy the superposition principle.
- There are waves with dispersive features $\omega(k)=\omega$.
- Group velocity of a wave is: $v_g=\frac{\partial\omega}{\partial k}=\frac{\partial E}{\partial p}$. Phase velocity is defined as $v_{ph}=\frac{\omega(k)}{k}$.
- For light:

$$v_g=\frac{c}{n+\omega\frac{\partial n}{\partial \omega}}=\frac{c}{n-\lambda_0\frac{\partial n}{\partial \lambda_0}}=v_p\left(1+\frac{\lambda}{n}\frac{\partial n}{\partial \lambda}\right)=v_p-\lambda\frac{\partial v_p}{\partial\lambda}=v_p+k\frac{\partial v_p}{\partial k}$$

16 Electric Field and Forces

16.1 Electric Force and Field

Coulomb force and field

- Coulomb force: $\vec{F}_C = K_C \frac{Qq}{r^2}\vec{u}_r = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2}\vec{u}_r$.
- Coulomb field: $\vec{E} = \frac{\vec{F}_C}{q} = \frac{K_C Q}{r^2}\vec{u}_r$.
- Coulomb potential energy and potential: $E_p(el) = \frac{KQq}{r}$, $V_e = \frac{K_C Q}{r}$.
- Coulomb constant: $K_C = 9 \cdot 10^9 Nm^2/C^2 = \frac{1}{4\pi\varepsilon_0}$. $\varepsilon_0 = 8.85 \cdot 10^{-12} C^2/Nm^2$.
- Multiple charges (superposition principle): $\vec{E} = \sum_{i=1}^{n} \frac{K_C Q_i}{r_i^2}\vec{u}_{r_i}$.
- Charge quantization: $Q_t = nq$, $n \in \mathbb{Z}$.
- Charge conservation: $\sum_i q_i(0) = \sum_j Q_j(f)$.
- Electric force can be repulsive and attractive (two types of charges).
- Electric flux: $\phi_E = \vec{E} \cdot \vec{S} \rightarrow \phi_E = \int \vec{E} \cdot d\vec{S}$.
- Gauss-Ostrogradskii theorem: $\phi_e = 4\pi K_C Q(\Sigma) = \frac{Q(\Sigma)}{\varepsilon_0}$.

Gauss theorem applications

- Electric field due to linear charge distribution: $\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 r}\vec{u}_r$.
- Electric field due to a plane sheet (infinite): $\vec{E} = \frac{\sigma}{2\varepsilon_0}\vec{u}_\perp$.
- Electric field between conducting infinite plates: $\vec{E} = \frac{\sigma}{\varepsilon_0}\vec{u}_\perp$.
- Electric field inside a conductor: $\vec{E} = \vec{0}$.
- Electric field due to a thin spherical shell outside the shell: $\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}$. Inside field: $E = 0$. Surface field: $E = \frac{Q}{4\pi\varepsilon R^2}$.
- Electric field due to uniformly charged spherical shell: $\vec{E} = \frac{Q}{4\pi\varepsilon_0}\vec{u}_r$ outside. Inside: $\vec{E} = \frac{Q}{4\pi\varepsilon_0 R^3}\vec{u}_r = \frac{\rho r}{3\varepsilon_0}\vec{u}_r$. On the surface, $\vec{E} = \frac{Q}{4\pi\varepsilon_0 R^2}\vec{u}_r$.

16.2 Dipoles and other formulae

Other formulae

- Superposition principle: $\vec{E} = \sum_{i=1}^{n} \vec{E}_i$. $V = \sum_{i=1}^{n} V_i$.
- Continous distributions: $\vec{E} = k \int \frac{dQ}{r^3} \vec{r}$.
- Electric dipole: $\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{2p}{r^3} \vec{r}$, $p = q\vec{r}$.
- Electric field due to dipole at equatorial position:

$$E = \frac{1}{4\pi\varepsilon_0} \frac{p}{r^3}$$

- Torque on an electric dipole placed inside a electric field: $\tau = \vec{p} \times \vec{E} = pE \sin\theta$.

17 Electrostatic potential and capacitance

Units of capacitance and dielectrics

- V is measured in volts.
- Conductors are materials where electrons or charges can freely move (electrons opposite to field, positive electrons along the field).
- Insulators: materials in which electrons are tighly bounds and when exposed to a field, electrons do not move.
- Electric fields inside conductors are zero.
- Electric field is perpendicular to charged surfaces.
- Capacitance: $C = \frac{Q}{V}$. Units: C/V=F=faradays.
- Energy stored by capacitor: $U_C = \frac{CV^2}{2} = \frac{Q^2}{2C} = \frac{1}{2} QV$.
- Dielectrics are electrical insulators that can be polarized.
- Capacitance of parallel plates: $C = \frac{K\varepsilon_0 A}{d}$. K is the dielectric constant.
- Capacitance when material is inserted into parallel plates:

$$C = \frac{K\varepsilon_0 A}{Kd - x(K-1)}$$

  where x is the thickness of the slab inserted.
- Spherical capacitor $C = \frac{4\pi\varepsilon_0 R_1 R_2}{R_2 - R_1}$. Isolated sphere has $C = 4\pi\varepsilon_0 R$.
- Polarisation: $\vec{P} = \varepsilon_0 \chi_e \vec{E} = \varepsilon_0(\varepsilon_r - 1)\vec{E}$.
- Electric displacement: $\vec{D} = \varepsilon_0 \vec{E} + \vec{P} = \varepsilon_0 \varepsilon_r \vec{E}$

Capacitor association

- Series capacitor: $\frac{1}{C} = \sum_{i=1}^{n} \frac{1}{C_i}$.
- Parallel capacitor: $C = \sum_{i=1}^{n} C_i$.

Energy

- Potential energy and potential: $U_e = q\Delta V$ (external). $\Delta U = -q\Delta V$ (field).
- Potential energy for dipoles: $U = -\vec{p} \cdot \vec{E}$.

18 Electric current and resistance

Electric current

- Electric current is the flow of charge through area or equivalently the change of charge with time:

$$I = \frac{dQ}{dt}, \quad I = \int \vec{J} \cdot \vec{S}$$

- Current density: $\vec{J} = \sigma \vec{E}$ (Ohm's Law).
- Drift velocity (free electrons): $V_d = \frac{J}{ne} = \frac{I}{neA}$.
- $\mu = \frac{V_d}{E} = \frac{e\tau}{m}$, with τ de average collision time.
- Macroscopic Ohm's law: $V = IR$, R is the electric resistance in ohms Ω.
- Resistance as function of resistivity: $R = \frac{\rho l \tau}{A}$. $\rho = \frac{1}{\sigma}$ in $\Omega \cdot m$. σ is the conductivity and ρ is resistivity.

Resistor association

- Series resistor: $R = \sum_i R_i$.
- Parallel resistor: $\frac{1}{R} = \sum_i \frac{1}{R_i}$.
- Resistance as function of temperature: $R = R_0(1 + \alpha\Delta T$. α is resistivity coefficient.
- Wheastone bridge: $\frac{R_1}{R_3} = \frac{R_2}{R_4}$ implies $V_c = V_D$.
- Meter bridge: $S = \frac{R(100 - L)}{L}$.
- Kirchoff's laws: 1) Loop law: $\sum_L V = 0$ for all loops, 2) Junction law: $\sum I(in) = \sum I(out)$ or $\sum_\gamma I(\gamma) = 0$.
- Generalized Ohm's law: $\Delta V = E - ir = i(R - r)$.

19 Magnetism and moving charges

Magnetism

- Magnetic field around certain materials implies a magnetic force.
- $\vec{F}_m = q\vec{v} \times \vec{B}$.
- $\vec{B}$ is measured in teslas (T), i.e., Wb/m^2. 1 gauss: $1G = 10^{-4}T$.
- Cyclotron: $F_m = F_c$. $R_c = \frac{mv}{qB}$. Period: $T_c = \frac{2\pi m}{qB} = \frac{1}{f_c}$.
- For noncircular trajectory, helicoidal motion is possible: $F_m = qvB\sin\theta$.
- Ampére's law: $\oint \vec{B} \cdot d\vec{r} = \mu_0 I$.
- Biot-Savart law: $d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{r} \times \vec{u}_r}{r^2}$. Also: $\mu_0 = \frac{1}{c^2\varepsilon_0}$.
- Maximum energy gained to accelerate a charged particle (excepting electrons):

$$\Delta E = \frac{q^2B^2p^2}{2m}$$

Magnetic field of important systems

- Finite wire: $B = \frac{\mu_0 I}{4\pi d}(\sin\theta_i + \sin\theta_2)$, θ_1, θ_2 are the angles formed with the lower and upper ends respectively.
- Circular wire (radius a): $B = \frac{\mu_0 I a^2}{2(a^2 + d^2)^{3/2}}$. At the centre: $B = \frac{\mu_0 I}{2a}$. Far from centre: $B = \frac{\mu_0 i a^3}{2d^3}$.
- Infinite straigth wire: $B = \frac{\mu_0 I}{2\pi r}$.
- Magnetic field of a toroid: $B = \frac{\mu_0 N I}{2\pi r}$. N is the number of turns, and I the current.
- Magnetic field at a point inside a long solenoid: $B = n\mu_0 I_e$. At the ends of the solenoid: $B = \frac{\mu_0 n I}{2}$. $n = N/L$ is the number of turns per unit length.
- Laplace law for current along conductors: $\vec{F} = I\int d\vec{l} \times \vec{B} = I\vec{L} \times \vec{B}$, where the last step is valid for finite conductors. Parallel conductors exert a force: $F(12) = \frac{\mu_0 I_1 I_2}{2\pi d}$.
- Magnetic torque: $\vec{\tau} = \vec{M} \times \vec{B}$. Magnetization: $\vec{m} = I\vec{S}$.
- $\mu_0 = 4\pi \cdot 10^{-7} N/A^2$.

20 Magnetism and matter

Magnetic substances

- Paramagnetism: tendency to increase the magnetic field due to magnetization; e.g., Al, Mn,...$\mu_r > 1$.
- Diamagnetism: tendency of strong magnetization in the direction of magnetic field; e.g., Fe, Co, Ni $\mu_r, \chi_m >> 1$.
- Ferromagnetism: tendency to magnetise in a direction opposite to the direction of magnetid field; e.g. Bi, Cu, Hg, Ni. $\mu_r, \chi_m < 1$.

Magnetic terms

- Magnetic intensity: $H = \dfrac{B}{\mu_0}$.
- Magnetic permeability: $\mu = \mu_0(1 + c_m)$.
- Magnetic susceptibility: $\chi = \dfrac{I}{H}$.
- Intensity of magnetization: $I_M = \dfrac{M}{V}$.
- Time period oscillating bar magnet: $T = 2\pi\sqrt{\dfrac{I}{MB}}$.
- Magnetic torque: $\vec{\tau}_m = \vec{M} \times \vec{B} = MB\sin\theta$. $M = 2ml$.
- Magnetic energy: $U_M = -\vec{M} \cdot \vec{B}$.
- $\vec{B} = \mu_0(\vec{H} + \vec{M})$. $\vec{M} = \mu_0(1 + \chi_m)\vec{H}$

Magnetic dipoles

- Dipole magnetic moment: $m\vec{u} = I\vec{S} = \dfrac{ev}{2\pi R} \cdot \pi R^2 = \dfrac{evR}{2}$. Also, $\vec{\mu} = -\dfrac{e\vec{L}}{2m_e}$ and $\mu = md$.
- Solenoid: $\mu = NIA$.
- Magnetic field for a dipole: $B = \dfrac{\mu_0 2M}{4\pi d^3}$. If $d >> l$, $B = \dfrac{\mu_0 M}{4\pi d^3}$.

Terrestrial magnetism

- Earth's is a natural source of magnetic field having geometric North and geometric South poles.
- Declination: Angle between magnetic meridian and geographical meridian.
- Inclination or dip δ: angle made by Earth's magnetic field with horizontal in the magnetic direction. $\delta(equator) = 0°$. $\delta(pole) = 90°$.
- Magnetic field lines are closed (no magnetic pole has been observed).

21 Electromagnetic induction

Faraday's law

- Whenever magnetic flux through an area bounded by a closed conducting loop changes, an emf (electromotive force) is produced in the loop.
- The emf is given by $\varepsilon = -\frac{d\phi_m}{dt}$, where $\phi_m = \int \vec{B} \cdot d\vec{S}$ is the magnetic flux.
- For ideal systems (ohmic conductors): $\varepsilon = IR$. For general circuits: $I = \frac{Blv}{r+R}$, whenever the system moves at constant or uniform speed (acceleration).
- Lenz's law: the sense of induced current is that oppositing the increase or decrease of magnetic flux.
- Thermal power (Joule's law): $P = \frac{v^2B^2L^2}{R} = I^2R$.

Mutual induction and self-inductance

- $\phi_m = MI$. $\frac{d\phi}{dt} = -M\frac{dI}{dt}$. Mutual induction: $M_{12} = \mu_0 N_1 N_2 \pi r_1^2 L$.
- AC generator $E = NBS\sin(\omega t + \varphi)$.
- Self-inductance: $L = \mu_0 n N^2 \pi r^2 L$.
- n is the turns per unit length.
- r is the radius of each loop.
- Growth of current: $I(t) = I_0(1 - e^{-t/r})$, where $r = L/R$. Decay of the current $I(t) = I_0 e^{-t/r}$.
- Energy stored in an inductor $U_L = \frac{1}{2}LI^2$

22 Alternating current

Transformers

- Turnsratio: $\frac{E_S}{E_P} = \frac{I_P}{I_S} = \frac{N_S}{N_P} = K$.
- Transformer efficiency about 99% can be reached.
- AC generator is any device generating alternating current $I = V/R = NBA\omega\sin(\omega t/R)$.
- AC current: $I = I_0 \sin\omega t$, $V = V_0 \sin\omega t$ is the AC voltage.
- Power in AC circuit: $P = \frac{E_0 I_0}{2}\cos\phi$

AC circuits

- Inductive (L) circuit: $I = I_0 \sin(\omega t - \pi/2)$.
- Capacitive (C) circuit: $I = I_0 \sin(\omega t + \pi/2)$.
- Resistive (R) circuit: $I = I_0 \sin(\omega t)$.
- R-C circuit: $I = I_0 \sin(\omega t + \phi)$. Impedance: $Z = \sqrt{R^2 + X_c^2}$. $X_c^2 = 1/\omega^2 C^2$.
- L-C circuit: $I = I_0 \sin(\omega t \pm \pi/2)$, with $X = X_L - X_C$. $X_L = \omega L$.
- L-R circuit: $I = I_0 \sin(\omega t + \phi)$. Impedance: $Z = \sqrt{R^2 + X_L^2}$, $X_L = \omega L$. Also, $\tan\phi = \omega L/R$. $I_0 = E_0/Z$. Power factor: R/Z. Voltage is leading.
- R-L-C circuit: $I = I_0 \sin(\omega t \pm \phi)$, $Z = \sqrt{R^2 + (X_c - X_L)^2}$. $X = X_c - X_L = \dfrac{1}{\omega C} - \omega L$. $\tan\phi = 1/\omega C - \omega L$. $I_0 = E_0/Z$.

23 Electromagnetic waves

Electromagnetic waves and their properties

- They do not need any material medium for propagation.
- They travel with speed $c = \sqrt{\dfrac{1}{\varepsilon_0 \mu_0}}$.
- Electromagnetic waves are produced by accelerated charges.
- Electromagnetic waves are transversal waves and own polarisation.
- Oscillation of electric and magnetic fields are in phase, and their ratio is constant: $E = Bc$.
- Combination of mutually perpendicular transverse electric and magnetic fields.
- Electromagnetic waves are vacuum solution to Maxwell equations.
- In 1886, Hertz became the first person to transmit and receive controlled radio.
- $I = dq/dt$. If electric charge is conserved, Ampère's law must be modified to include the displacemente current:

$$I_d = \varepsilon_0 \frac{d\phi_e}{dt}$$

Maxwell equations

- Gauss law for electic field: $\oint \vec{E} \cdot d\vec{S} = Q/\varepsilon_0$ or $\nabla \cdot \vec{E} = \rho/\varepsilon_0$.
- Gauss law for magnetic fields: $\oint \vec{B} \cdot d\vec{S} = 0$, $\nabla \cdot \vec{B} = 0$.
- Faraday's law: $\oint \vec{E} \cdot d\vec{r} = -\dfrac{d\phi_m}{dt}$ or $\nabla \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t}$.
- Generalized Ampère's law: $\oint \vec{B} \cdot d\vec{r} = \mu_0 \left(I + \varepsilon_0 \dfrac{d\phi_e}{dt} \right)$ or $\nabla \times \vec{B} = \mu_0 \vec{J}$, with $\vec{J} = \vec{j} + \vec{j}_d$

Electromagnetic spectrum

- Radio waves for $\lambda > 10^8 nm$, microwaves for $10^8 nm \lambda > 10^5 nm$, infrared (IR) for $10^5 nm > \lambda > 700 nm$.
- Visible light for $700nm > \lambda > 400nm$. UV rays for $400nm > \lambda > 10nm$, X-ray for $10nm > \lambda > 0.01nm$, γ-rays for $0.01nm > \lambda$.

EM energy

- EM energy density: $U = \frac{\varepsilon_0 E^2}{2} + \frac{B^2}{2\mu_0}$.
- Average over long time: $U = \frac{\varepsilon_0 E^2}{2} = \frac{B^2}{2\mu_0}$

24 Ray optics and optical instruments

Reflection and refraction laws

- Reflection law: $\theta_i = \theta_r$
- Refraction law (Snell's law): $n_i \sin\theta_i = n_r \sin\theta_r$. Also: $\frac{\sin\theta_i}{v_i} = \frac{\sin\theta_r}{v_r}$.
- Limit angle, total internal reflection: $\sin^{-1}\theta_L = \frac{n_r}{n_i}$.
- Refraction index: $n = \frac{c}{v}$.

Lenses and mirrors

- We use DIN sign conventions.
- Spherical dioptrics: $\frac{n_2 - n_1}{R} = \frac{n_2}{s'} - \frac{n_1}{s}$.
- Lens maker's formula: $\frac{1}{f'} = \frac{n_2 - n_1}{n_1}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$.
- Thin lens formula: $\frac{1}{f'} = \frac{1}{s'} - \frac{1}{s}$. Lateral magnification: $\beta = \frac{y'}{y} = \frac{s'}{s}$.
- Spherical mirrors: $\frac{2}{R} = \frac{1}{f} = \frac{1}{s'} + \frac{1}{s}$. Lateral magnification: $\frac{y'}{y} = -\frac{s'}{s} = \beta$.
- Plane mirrors: $s' = -s$. Lateral magnification $\beta = 1$.
- Focal power: $P = \frac{1}{f}$ in dioptries. $1D = 1m^{-1}$. Combined power when lenses are in contact: $P = 1/f_1 + 1/f_2$. Combinaed power when lenses are separated: $P = 1/f_1 + 1/f_2 - \frac{d}{f_1 f_2}$.
- Near point for eyes: 25 cm.
- Defects of vision: astigmatism, myopia, hypermetropia, tired vision (presbicia).

25 Wave optics

Interference and polarisation

- Two waves superimpose to form a wave greater or lower or same amplitude. $I = (\sqrt{I_1} + \sqrt{I_2})^2$. Constructive: $A = A_1 + A_2$. Destructive: $A = A_1 - A_2$.
- Doppler effect (non-relativistic): $\frac{\Delta f}{f_0} = -\frac{v_r}{c}$.
- Polarisation (when waves are transverse).
- Brewster's law: $\mu = \tan\theta_p$. θ_p is the polarisation angle.
- Malus law: $I = I_0 \cos^2\theta$.
- Spherical wavefronts: $I \propto 1/r^2$ and $A \propto 1/r$.

Diffraction and principles

- Resolving power: $\theta = 1.22\frac{\lambda}{D}$.
- For microscopes: $P = 2n\sin\theta/\lambda$.
- Huygens principle: every point on the primary wavefront is the source of secundary waves.
- Coherent sources: any 2 sources are said to be coherent if the initial phase difference remains constant in time, otherwise are incoherent.
- Diffraction by single slit: $b\sin\theta = n\lambda$ (dark fringes) are lienar widht of central maxima. Width of central maxima=$2\lambda/b$. Angular width is $\theta = 2D\lambda/b$. $b\sin\theta = (2n+1)\lambda/2$ are maxima for bright fringes.
- Young's experiment: fringe width equals $\beta = D\lambda/d$ for 2 consecutive fringes. Distance between nth bright fringe and central fringes is $x_n = n\lambda D/d$, D is the distance between source and screen, and d the separation between the 2 slits.
- Young's experiment: distance between nth dark fringe and central fringe is $x_{nc} = \frac{(2n-1)\lambda D}{2d}$. Path difference for bright fringe is $\Delta\phi = n\lambda$ and for dark fringes $\Delta\phi = \left(n + \frac{1}{2}\right)\lambda$.

26 Special Relativity

Lorentz transformations and SR postulates

- SR is based in the constancy of speed of light in vacuum and the universality of mechanical and electromagnetical laws.

- Lorentz transformations:

$$x' = \gamma(x - vt) \tag{5}$$

$$y' = y \tag{6}$$

$$z' = z \tag{7}$$

$$t' = \gamma(t' - \frac{vx}{c^2}) \tag{8}$$

- Simultaneity is relative to observers. There is no universal time. Time is relative to observers.

- Length contraction: $L' = L_0\gamma^{-1}$.

- Time dilation: $\Delta t = \gamma \Delta t'$.

- Velocities, even when relative, can not surpass the speed of light: $V = \dfrac{V_1 + V_2}{1 + \dfrac{V_1 V_2}{c^2}}$.

- Energy is a form of mass and viceversa: $E = mc^2$.

- Momentum is not linear with velocity $p = m\gamma v$.

- Energy-momentum dispersion relation in SR: $E^2 = (pc)^2 + (mc^2)^2$.

- Geometry is not euclidean in space-time but hyperbolic: $\Delta s^2 = \Delta x^2 - c^2 \Delta t^2$.

- Maxwell equations are invariant under Lorentz transformations but not the newtonian laws.

- In the low velocity regime, we recover newtonian mechanics.

- $\gamma = \left(1 - \dfrac{v^2}{c^2}\right)^{-1/2}$ is the relativistic gamma factor, and generally we also introduce $\beta = v/c$.

- If $E = mc^2 \cos\varphi$ and $pc = mc^2 \sin\varphi$, then $pc/E = \tanh\varphi$. If $E = m\gamma c^2$ is the total energy, then $\gamma = \cosh\varphi$ and $\varphi = \cosh^{-1}\gamma$, $v/c = \tanh\varphi$, so $\beta = \tanh\varphi = \tanh(\cosh^{-1}\gamma)$.

- Newton approximation is useful: $(1 + x)^n \approx 1 + nx$.

27 General Relativity and Cosmology

Hubble's law

- At the beginning of the 20th century, E. Hubble discovered that the Universe is expanding: $v = HD$. H is the Hubble constant. $t_U = 1/H$. $R_U = c/H$.
- Einstein's special relativity implied that gravity should be relativized, since Newton's gravity was not consistent with SR.
- EFE (Einstein's field equation for gravity): $G_{\mu\nu} + \Lambda g_{\mu\nu} = \dfrac{8\pi G}{c^4} T_{\mu\nu}$.
- EFE implied for simplified models the Friedmann's equations and that the Universe would expand.
- In 1998 dark energy, and the cosmological constant, was probed to be non-zero.
- The Hubble parameter is really non-constant.
- Critical density: $\rho_c = \dfrac{3H_0^2}{8\pi G}$.
- The Universe is described by LCDM today, with $\Omega_{DM} \sim 0.25, \Omega_\Lambda \sim 0.7, \Omega_m \sim 0.05$.
- GR predicts: gravitational waves, gravitational time-delay beyond the one in SR, black holes, Mercury's precessions of the perihelion, gravitational lensing, the Big Bang, and many other non-classical newtonian gravitational effects like the Lense-Thirring effect, gravitomagnetism, and more. Pushed to the extreme, GR predicts space-time singularities.

28 Waves-particle duality and Quantum Physics

Quantum hypothesis

- Planck quantum hypothesis was necessary to solve the blackbody radiation problem. $I = \sigma T^4$.
- Light or radiation must be quantized $E = nhf = \dfrac{nhc}{\lambda}$.
- Photoelectric effect vindicates light behaves like quanta not wavy to some extent.
- Einstein equation for photoelectric effect: $hf = hf_0 + E_c(m)$. $W_e = hf_0$ is the extraction function or work function for a metal. $E_c = mv^2/2$ is the maximal kinetic energy of the electron.
- $E_c(m) = eV_f$ and $hf_0 = eV_0$.
- Threshold frequency f_0 and wavelength $\lambda_0 = c/f_0$. There is no photoelectric effect below f_0 or above λ_0.
- Wave-particle duality (probed in DavissonGermer experiment): $\lambda = h/p$. For electrons in a potential, we get
$$\lambda = \frac{1.227}{\sqrt{\Delta V}} nm$$
and that is the principle of electron microscope or alike.
- Also: $\lambda(dB) = \dfrac{h}{\sqrt{2mK_m}} = \dfrac{h}{\sqrt{2me\Delta V}}$ for non-relativistic electrons. Relativistic versions are known to be observed and be fully valid.
- Photocurrent is a complex function $I = I(V)$.
- Plotting K_m or V_f as funcion of frequency provides linear graphs, contrary to classical expectations.

29 Atoms

Bohr model

- Limitations: a single electron, do not explain Zeeman's or Stark's effects. Do not explain spin or multielectron atom spectra.
- Hypotheses: Planck quantization for light, stationary circular orbits and quantization of angular momentum. Consequences: kinematical quantization of radius, energy and other variables.
- $r_n = a_0 \frac{n^2}{Z}$.
- $v_n = \frac{\alpha Z c}{n}$.
- $E_n = -\frac{Z^2 Ry}{n^2}$.
- $\Delta E = Ry\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$. $Ry = 13.6eV = 2.18 \cdot 10^{-18}J$.
- Lyman series (UV): $n_i = n_1 = 1$.
- Balmer series (visible): $n_i = n_1 = 2$.
- Paschen series (IR): $n_i = n_1 = 3$.
- Brackett series (IR): $n_i = n_1 = 4$.
- Pfund series (IR): $n_i = n_1 = 5$.

Experiments

- Rutherford dispersion experiment: uses the impact parameter $b = \frac{Ze^2 \cot(\theta/2)}{4\pi\varepsilon_0(mv^2/2)}$
- Atoms have parts: nuclei and shells.
- Do not explain stability. Do not explain spectra.
- Sizes of the nuclei: $d \sim 1fm$. $1fm = 10^{-15}m$.

30 Nuclei

Decay law

- Decay law: $N(t) = N_0 e^{-\lambda t}$. Equivalently: $m(t) = m_0 e^{-\lambda t}$. Also. $N_0 - N = N_0(1 - e^{-\lambda t}$ are the decayed atoms. Moreover: $N(t) = N_0 2^{-t/T_{1/2}}$.
- Decay constant: $\lambda = \frac{\ln 2}{T_{1/2}} = \frac{1}{\tau}$, where τ is the mean lifetime. $T_{1/2}$ is the halflive time.
- Activity: $A(t) = A_0 = |dN/dt| = \lambda N$. Units: $1Bq = 1s^{-1}$.
- $\frac{dN}{dt} = -\lambda N$.
- Mass defect: $\Delta m = Zm_p + Nm_n - m_{nuc}$.
- Binding energy per nucleon: $\Delta E/A = \Delta mc^2/A$, $A = Z + N$.

Nuclei and particle properties

- Nuclei are bond by nuclear forces.
- Weak and strong forces act beyond electromagnetic forces.
- Protons and neutrons are not fundamental.
- Yukawa potential described nuclear interactions at effective level (due to meson interchangers, e.g., pions).
- Hadrons feel strong (weak and electromagnetic forces). Hadrons can be baryons or mesons.
- Neutrinos and leptons beyond the first family are found too.
- Decays are due to weak force and changes of flavor. Electroweak interactions are mediated by W, Z, bosons beyond photons. Massive vector gauge bosons imply the existence of a scalar boson, the mass giver Higgs boson.
- Nuclei described as a drop have $R = R_0 A^{1/3}$ typical size, and big densities.
- Basic types of decay include alpha, beta, gamma or capture reactions.
- Continuous spectrum of beta decay implies the existence of the neutrino.
- Neutrinos have 3 flavors, leptons have 3 flavors, quarks have 6 flavors.

31 Subatomic particles and Standard Model

Standard model

- Standard Model describes the strong, weak and electromagnetic interactions of fundamental particles.
- It requires a scalar field, the Higgs field, whose quanta have non-zero value and are the mass-givers of the elementary particles (not the composite particles).
- QED describes electromagnetism at low energies, but it requires the electroweak interaction and the SM at energies about 80 GeV or higher. At those energies, the W and Z bosons appear.
- At $M_H = 125GeV$ we find the Higgs particle, and at about $m_t = 171GeV$ the top quark, the heaviest particle in the SM.
- Neutrinos are the most mysterious part of the SM. They oscillate and transform between mass eigenstates. Neutrinos could be Dirac or Majorana particles.
- Gravity is not included in the SM.
- Fundamental leptons: $e, \mu, \tau, \nu_e, \nu_\mu, \nu_\tau$.
- Fundamental quarks: u, d, c, s, t, b.
- Gauge bosons: W^+, W^-, Z^0, γ, g. Also we have scalar Higgs bosons H_0.

32 Semiconductor electronics

General electronics

- Heitler and London discovered in 1927 the energy bands.
- Energy bands are a range of energies associated with the quantum states of electrons in crystalline solids.
- Energy bands that are completely filled with electrons at zero kelvin are valence bands.
- Conduction bands are bands with higher energy above the valence band and a gap $\Delta E_g = E - E_F$.
- The difference between the highest energy in a valence band and lowest energy in the next higher band are the so-called band gap or forbidden gap.
- Conductors are materials with $E_g = 0$, and they have density of charge carries high, about $n/V = 9 \cdot 10^{23} m^{-3}$.
- Classification of metals and semiconductors or insulators are based on band theory.
- Semiconductors are materials with $E_g = 0.72 eV$ like Ge, or $1.1 eV$ for silicon.
- For n-type $n/V = 7 \cdot 10^{15} m^3$,like doped Si with P. For p-type, $n/V = 1 \cdot 10^9 m^{-3}$, like for Si doped with Al.
- Insulators have $E_g = 7eV$ or more.

Semiconductor theory

- Intrinsic semiconductor: charge carrier concentration $n_i = n_e = n_h$. Example: silicon as pure semiconductor.
- Extrinsic semiconductor: impure or doped can be P-type semiconductors or N-type semiconductors.
- P-type extrinsic semiconductors: Si or Ge doped with trivalent (B,Al) elements. Electrons are minority carriers. Holes are majority carriers.
- N-type extrinsic semiconductors: Si or Ge doped with pentavalent (P,As,Sb) elements. Electrons are majority carriers. Hole are minority carriers.

Semiconductor diodes

- Forward biased p-n. Junction diode (made of an atomic level N-type and P-type semicondcutors): +ve terminal to P-side, -ve to N-side, diffusion current increases, and depletion layer reduced.
- Reverse biased p-n: -ve terminal to P-side, -ve terminal to N-side, diffusion current decreases, depletion layer increases.
- LEDs: light emitting diodes. Used in TV and electronic devices.
- Photodiodes: p.n. junction whose function is controlled by the light allowed to fall on it.
- Zener diode: used as a voltage regulator.

33 Quantum Physics extras

Quantum physics principles

- Particles and waves have dual nature. They must be described by wavefunctions.
- Wavefunctions satisfy the Schrödinger equation $H\Psi = E\Psi$.
- Wavefunctions are not measurable, they are interpreted as probability amplitudes.
- $|\Psi|^2 = \Psi^*\Psi$ is a probability density.
- Heisenberg's uncertaingy principle: there are limits to simultaneous measurement of certain magnitudes. For instance:

$$\Delta x \Delta p \geq \hbar/2 \quad \Delta E \Delta t \geq \hbar/2$$

- Wavefunctions are complex numbers. They can also be imagined as vectors in abstract spaces.
- Quantum computing explore the superposition principle and quantum nature of reality. Qubits, qutrits, qudits,...,quits (quantum fields) are examples of this nature.
- Quantum Physics and Quantum Field Theory like the SM are compatible with SR and Quantum Mechanics principles.
- Quantum Mechanics implies certain non-classical correlations for composited systems, called entangled states. Moreover, there are other non-classical correlation measurements that are observable, like quantum discord.